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# NEARFIELD ACOUSTICAL HOLOGRAPHY IN A REFLECTIVE MEDIUM USING AN ITERATIVE REGULARIZATION METHOD

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## INTRODUCTION

Nearfield Acoustical Holography (NAH) is conceptually a powerfull tool for the extrapolation of fields radiated by sound sources. The Fourier method allowed its numerical implementation for numerous applications [1], especially for the inverse problem of sound source reconstruction. This reconstruction consists in a back-propagation processing on the wave-number components of measurements on a surface in the nearfield of the sources.

Measurements need theorically to be noise-free. This limits NAH to laboratory applications where the influence of the noise can be controled. In practice, only rough free-field situations can be tolerated for measurements. Otherwise, results of the reconstruction become physically meaningless.

In a reflective medium, the Fourier method becomes unexploitable if the influence of noise is neglicted in the back-propagation process. A residual noise may enhance the inherent unstability of the back-propagation process. This express a case of ill-posed inverse problem.

For highly reflective situations, a double-layer separation process [2],[3] can be used. However, some residual noise may be tolerated in the data if the unstability of the back-propagation is controlled. For this purpose, low-pass filtering in the wave-number domain is usually applied [4]. It can be shown to be connected with a standard regularization method [5]. Nevertheless, the usual question remains : how to choose the filter and its shape parameters ? Iterative inversion techniques circunvent this difficulty.

## THEORICAL FORMULATION OF THE FOURIER METHOD

At a given frequency of interest $f_0$, the analytical expression of the complex pressure field $P_m(r,k_z)$ in K-space corresponds to the cylindrical measurement enveloppe of complex pressure field $p(r,\theta,z)$ in the space domain. $P_m(r,k_z)$ components are given by :

$$P_m(r\,,k_z) = B_m(k_z)Z_m(k_a.r) + N_m(r\,,k_z) \qquad (1)$$

where $N_m(r\,,k_z)$ are residual noise components at radius r, and $B_m(k_z)$ are the unknown coefficients of elementary radialy divergent waves of the radiating source. In the previous expression, $k_a$ and $Z_m$ are given by :

• $Z_m(k_a.r) = H_m^{(1)}(k_r.r)$ Hankel function of $1^{st}$ type

for $k^2 > k_z^2$ $k_a = k_r = \sqrt{k^2 - k_z^2}$ (propagative waves)

• $Z_m(k_a.r) = K_m(k_r'.r)$ Modified Bessel's function of $2^{nd}$ type.

for $k^2 < k_z^2$ $k_a = k_r' = \sqrt{k_z^2 - k^2}$ (evanescent waves).

Measurements of spatial pressure $p(r_H,\theta,z)$ lead to a spectral estimation of $P_m(r_H,k_z)$, performed with FFT. Neglecting the noise $N_m(r\ ,k_z)$, it is theorically possible to extract the solution $B_m(k_z)$ for each set of K-space components $(m,k_z)$. This is the standard Fourier method, consisting in a direct deconvolution of the propagation effects on wave-number components in K-space :

$$P_m(r_S\ ,k_z) = P_m(r_H\ ,k_z)\ .Z_m(k_a\ .r_S)\ /\ Z_m(k_a\ .r_H) \qquad (2)$$

Performing inverse FFT on expression (2) lead to the reconstruction of spatial pressure $p(r_S,\theta,z)$ at a radius $r_S$ close to the source. The unknown coefficients $B_m(k_z)$ are often mesestimated because of the neglected noise quantities $N_m(r_H,k_z)$ (coresponding to numerical or instrumental noise, as well). By the fact, the numerical processing is unstable and brings unexploitable spatial results. This is a typical effect of an ill-posed inverse problem.

## REGULARIZATION BY USE OF THE REBLURRING METHOD

The reblurring method is a direct search method wich does not require any windowing [6]. This direct search formulation avoids the unstability caused by the direct inversion method. An initial guess solution activates the iterative method. This solution is forward propagated to the hologram surface. An additive correction is calculated from the tentative image and the measured image. Then, the solution is updated with this correction before a new iteration. For each set $(m,k_z)$, the global formulation is

$$P^\wedge_{k+1} = P^\wedge_k + ß.R.(M-T.P^\wedge_k) \qquad (3)$$

where

| | |
|---|---|
| $P^\wedge_k$ | is an estimation of the solution $P_m(r_S,k_z)$ at the iteration order k, |
| ß | is an ajustement parameter, |
| R | is a feedback term : R is the complex conjugate of T, |
| M | is the measurement component $P_m(r_H,k_z)$ at radius $r_H$, |
| T | is the propagative quantity given by : |

$$T = Z_m(k_a.r_H)\ /\ Z_m(k_a.r_S) \qquad (4)$$

In this method, the noise amplification is slower than the convergence to an acceptable solution. Therefore, the regularization consists in stopping the iterations before results tend to direct inversion solutions. The ajustement parameter ß is selected for each set $(m,k_z)$, so as to verify the convergence criterion $|1 - ß\ |T|^2| < 1$ Iterations are stopped after a stagnation of the residue $|\ M - T.P^\wedge_k\ |$ or after it falls below an arbitrary level $\varepsilon$.

## MEASUREMENTS

This iterative technique is now applied to measurements in a reflective environment. Measurements consists in a double set of 1/4" microphones moved in the nearfield of a a point-driven end-capped cylindrical shell excited at $f_0$=1425 Hz [7]. Using the excitation signal as a reference, it comes the complex pressure field to be processed with the iterative technique described above. The walls of the reverberation room give the external noise added to some noise generated with a loud-speaker directed on a remote wall of the room [3].

## EXPERIMENTAL RESULTS

We compare results of complex pressure reconstructions at radius $r_S$, in terms of reconstructed modal shapes (spatial pressure modulii) of the radiating shell. Direct back-propagation and low-pass filtering of measurements are shown at **figures** 1 and 2. Although they are filtered with an optimaly choosen low-pass filter, direct propagation results of perturbated data (**fig.**2) always show some important mesestimations (compare with the reference processing of free-field data (**fig.**1)). Results of an iterative inversion with the reblurring method are shown at **figures** 3 and 4. Iterative inversion of free-field data gives excellent solutions (**fig.**3). Solutions with the iterative inversion of perturbated data are close to the results of **figure** 2. Therefore, a lower arbitrary level $\varepsilon$ must be selected. It causes a reduction of the number of iterations and consequently of the amplification of noise. The new results (**fig.**5) look closer to reference results of **figure** 1.

## CONCLUSIONS

Regularization techniques are essential for a correction of the ill-posed formulation of NAH applications with the Fourier method. In this paper, an iterative inversion method was investigated to examine its effects on the stability of holographic reconstructions. This approch was found to be a profitable alternative to empirical standard regularizations with K-space windowing. Moreover, this technique allows a better control of the trade-off between stabilisation versus loss of resolution. Current research is directed towards extending this approach to a multidimensional iterative technique including spatial constraints.

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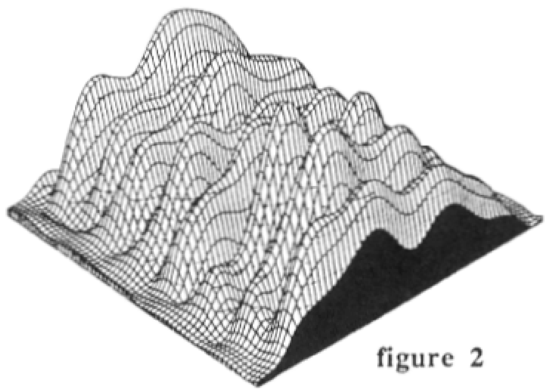


figure 2

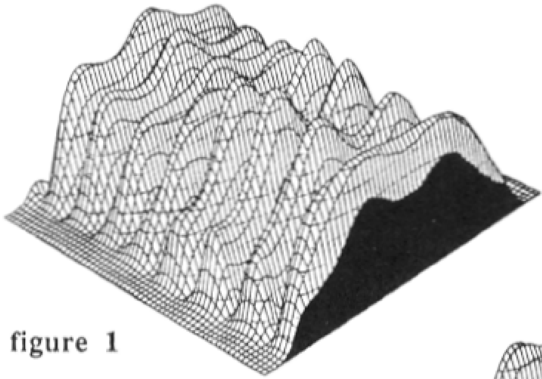


figure 1

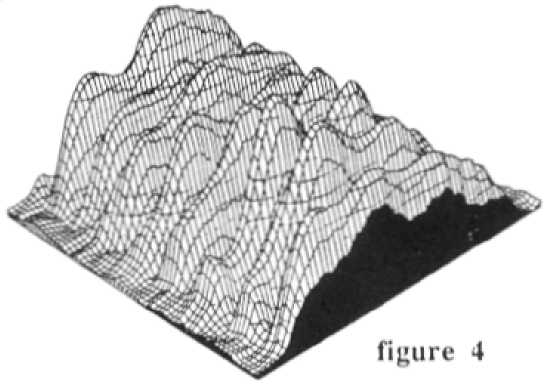
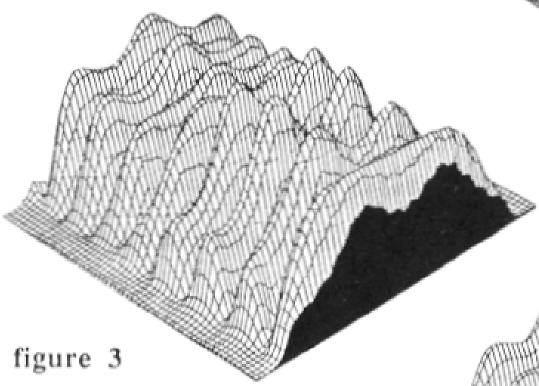


figure 4



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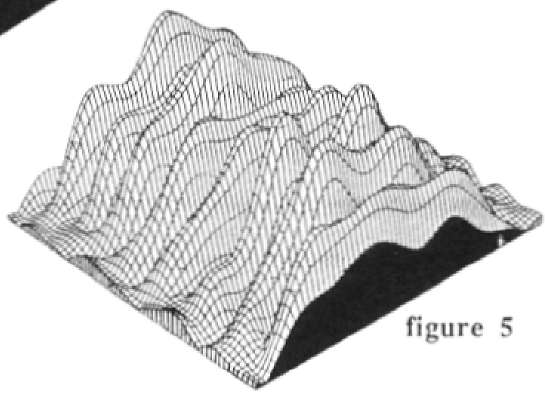

figure 5

Modulii of complex spatial pressure reconstructed at the surface of a radiating shell.
Figure 1 : direct inversion and K-space filtering on free-field components
Figure 2 : direct inversion and K-space filtering on noisy components
Figure 3 : iterative inversion on free-field components
Figure 4 : iterative inversion on noisy components
Figure 5 : iterative inversion on noisy components and ajustement of an iteration stopping criterion.