A MULTIFREQUENCY SCINTILLATION SENSING OF RANGE DEPENDENT CURRENT

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ABSTRACT

Transverse flow of inhomogeneous current produces fluctuation of the acoustic signal passing through it. These fluctuations vary with the signal frequency change due to variation of the Fresnel zone. Respectively, the fluctuations of signals at two different frequencies are coherent in a low frequency range of the spectrum and non-coherent in the high-frequency band when ocean inhomogeneous will be smaller then the difference in transversal dimensions of Fresnel area cross-sections for different frequencies. Coherence function of the signals at different frequencies depends on a current flow spatial distribution. Therefore multi-frequency sounding of the current leads to possibility retrieves the flow spatial profile.

INTRODUCTION.

Variation of sound signal propagated through the inhomogeneous ocean flow can be used to solve the inverse problem of the sound speed field and current velocity acoustic measurements. Spatially separated acoustical paths are applied to measure transverse current flow by using the correlation of acoustic scintillations recorded at transversely spaced receivers -- a technique that has been suggested for ocean current measurements in [1,2,3]. The essence of the technique lies in the interpretation of the combined spatial and temporal variability of propagated sound. Sound signal passing through the ocean fine structure is modulated, producing an irregular pattern of amplitude and travel time variations at the receivers. These variations evolve with the intervening medium changes. Under assumption that the fluctuations in the medium are produced mainly by the advection of a frozen fine structure field (Taylor's model of turbulence), evolution of the signal pattern can be directly linked to the motion of the medium. This allows determination of transverse component of the current by measurements of the time lag between the signal variations at two closely spaced receivers. In general, the coherence of sound received by different hydrophones (separated in space, time or frequency) in the ocean is a useful measure of the behavior of the intervening fluid medium [4]. Correspondingly, another version of the scintillation method is based on the use of a multi-frequency signal [5]. This method can be considered as a "frequency-domain" version of the conventional scintillation approach to the current velocity registration based on the measurement of the signal correlation transmitted from the source to the two separated receivers (space-domain scintillation), [1,2,3]. The transverse flow of inhomogeneous fluid produces fluctuation of the acoustic signal passing through it. These fluctuations vary with the CW signal frequency change due to variations of the Fresnel zone size. Respectively, the fluctuations of amplitude and phase of frequency-spaced signals are coherent at the low fluctuation frequency and non-coherent at the high-frequency band. The measurement of the

cutoff frequency allows determination of the transverse current velocity. The feasibility of this technique was confirmed in [6,7,8], and [9] for wind velocity measurement, in turbulent atmosphere.
In all papers mentioned above, the fine structure of refractive index fluctuations was assumed to be caused only by turbulence and was described by the isotropic Kolmogorov-Obukhov spectrum, and temporal variations were caused only by "frozen" transportation of inhomogeneities, along with the turbulent refractive index fluctuations (isotropic first of all). Here we will follow such an approach to consider the method to retrieve the current velocity spatial profile by multi-frequency sound signal propagated along the path between one source and one receiver. Therefore one could consider that method as a “one path tomography” approach for ocean flow spatial retrieval on the path between one pair of sound transducers.

GENERAL EQUATIONS

Let us consider the propagation through the random medium of a set of plane CW waves of different frequencies ω_n :

$$\varphi_n = \exp(\psi_n - i\omega_n t)\,. \tag{1}$$

Here, φ_n is the pressure or velocity potential, ψ_n is the complex phase,

$$\psi_n = \chi_n(t) + iS_n(t), \tag{2}$$

$\chi_n(t)$ and $S_n(t)$ are the log-amplitude and the phase of the signal, respectively.
In the framework of the smooth perturbation method, the fluctuations of the complex phase of the signal at the distance *L* from the source are [10]

$$\psi_n(t) = \frac{k_n^2 L}{2\pi}\int_0^L \frac{dx}{x(L-x)}\iint dydz\,\mu(x,y,z,t)\exp\left\{\frac{ik_nL(y^2+z^2)}{2x(L-x)}\right\}, \tag{3}$$

here, $k_n = \omega_n / c$ is the wave number, c is the sound velocity, and $\mu(x,y,z,t)$ are the refraction index fluctuations. The sound waves are thought propagate along the x-axis, which is a horizontal line connected to a point sound source ($x = 0$) and a non-directional receiver ($x = L$), the axis z directs vertically. The relative coherence of the signals can be characterized by the cross correlation function and its Fourier transform, i.e., the cross spectrum. The cross spectrum of the log-amplitude temporal fluctuations is

$$W_\chi^{(n.m)}(\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\nu\tau}\left\langle \chi_n(t)\chi_m(t+\tau)\right\rangle d\tau \tag{4}$$

and the cross spectrum $W_S^{(n.m)}(\nu)$ for the phase temporal fluctuations $S_n(t)$ and $S_m(t)$ can be introduced by the similar equation. The cross and power spectra defined by such a way depend upon the spectrum of the refraction index $\mu(\mathbf{R},t)$ for inhomogeneities in the medium that can be characterized by the spatial-temporal power spectrum density:

$$\Phi_\mu(\mathbf{q},\nu) = \frac{1}{(2\pi)^4}\int B_\mu(\boldsymbol{\rho},\tau)\exp\left[i(\nu\tau - \mathbf{q}\boldsymbol{\rho})\right]d^3\rho\, d\tau\,, \tag{5}$$

here 4-D integration in the infinite limits is implied, and $B_\mu(\boldsymbol{\rho},\tau)$ is the spatial-temporary auto-correlation function of the refractive index fluctuations:

$$B_m(\tilde{\mathbf{n}}, t) = \left\langle m(\mathbf{R}, t) m(\mathbf{R} + \tilde{\mathbf{n}}, t + t) \right\rangle . \qquad (6)$$

Here, the brackets <…> denote the statistical averaging. Spatial statistical homogeneity on a vector variable $\mathbf{R}(x, y, z, t)$ and temporal stability of refractive index fluctuations $m(\mathbf{R}, t)$ are implied as well. If the spatial-temporal fluctuations of the refractive index $m(\mathbf{R}, t)$ are the result of turbulence in the presence of a regular medium flow with a current velocity $\mathbf{U}$, then the spatial-temporal power spectrum density of the refractive index fluctuations can be represented in the form

$$\Phi_m(\mathbf{q}, n) = \Phi_m(\mathbf{q}) d(n - \mathbf{qU}) . \qquad (7)$$

If the current flow does not have a vertical component ($U_z = 0$), then the cross spectra for the phase $W_S^{(n.m)}(n)$ and log-amplitude $W_c^{(n.m)}(n)$ temporal fluctuations can be expressed as follows:

$$W_{c,S}^{(n.m)}(n) = \frac{pk^2}{2(1-\Omega^2)} \int_0^L dx \iint dq_y dq_z \Phi_m(0, q_y, q_z) d\left[n - U(x) q_y\right] \times$$
$$\times \left\{ \cos\left[x(L-x) \frac{q_\perp^2}{kL} \Omega \right] \mp \cos\left[x(L-x) \frac{q_\perp^2}{kL} \right] \right\} . \qquad (8)$$

Here, the upper sign (-) corresponds to the log-amplitude cross spectrum and the lower one (+) corresponds to cross spectrum of phase variations, $q_\perp^2 = q_x^2 + q_y^2$, $U(x)$ is the projection of current velocity on the axis y, Ω is the relative frequency spacing, and k is the median sound wave number:

$$\Omega = \frac{w_n - w_m}{w_n + w_m}, \qquad k = \frac{2k_n k_m}{k_n + k_m} . \qquad (9)$$

Here and below we assume, without losing generality, that $w_n \geq w_m$, and hence $\Omega \geq 0$. Let's consider an application of Eq. (9) for particular cases of medium motion with some current velocity profile $U(x)$ and spatial power spectra of refractive index fluctuations.

In the case of "frozen" (stable) inhomogeneities moving in the horizontal plane with a constant velocity $U(x)$, we can carry out the integration over q_y in (8) due to the delta-function $d\left[n - U(x) q_y\right]$. Let's come to dimensionless parameter of distance $z = \frac{2x - L}{L}$, and to dimensionless function of current velocity distribution $h = \frac{q_z U(z)}{n}$. If the main cause of the refractive index fluctuations is the turbulence (due, for example, to the current flow), we can use the Obukhov-Kolmogorov spatial power spectrum.

$$\Phi_m(q_\perp, q_z) = 0.033 C_m^2 \frac{1}{(q_\perp^2 + q_z^2)^{11/6}} , \qquad (10)$$

C_m^2 is the structure constant of the turbulence field. And equation (8) for this case takes the dimensionless form

$$W_{c,S}^{(n.m)}(\nu)=\frac{4\pi 0.033C_m^2k^2L}{(1-\Omega^2)}\left(\frac{L}{4k}\right)^{4/3}\int_0^1 dz\left[\frac{\nu_0 u(z)}{\nu}\right]^{8/3}\int_0^\infty\frac{d\eta}{(1+\eta^2)^{11/6}}\times$$
$$\times\left\{\cos\left[\frac{\nu^2}{\nu_0^2u^2(z)}\left(1+\eta^2\right)(1-z^2)\Omega\right]\mp\cos\left[\frac{\nu^2}{\nu_0^2u^2(z)}\left(1+\eta^2\right)(1-z^2)\right]\right\}. \quad (11)$$

Here we introduced the characteristic frequency $\nu_0=U_M\sqrt{k/L}$ (it is inverse to the travel time across Fresnel zone $\sqrt{L/k}$ by inhomogeneities traveling with the maximum of transversal velocity ($U_M=\max U(z)$) and $u(z)=U(z)/U_M$ is normalized current flow spatial profile. Note the result of the calculations is dependent on $U^2(z)$ and will be the same if we replace $z\to -z$. Another words turbulence flow affect sound signal crossed it in transversal direction symmetrically with respect to current direction and flow distance from center of the sound path. These cross-spectra will define the coherence function of log-amplitude and phase fluctuations on the carrier frequencies ω_m and ω_n

$$\Gamma_{c,S}^{(m,n)}(\nu)=\frac{W_{c,S}^{(n.m)}(\nu)}{\left[W_{c,S}^{(m.m)}(\nu)W_{c,S}^{(n.n)}(\nu)\right]^{1/2}}. \quad (13)$$

The coherence function $\Gamma_{l,S}^{(m,n)}(\nu)$ will oscillate due to term $\cos\left[\Omega\nu^2/\nu_0^2\right]$ in integral. The structure of the cosine argument indicates that pronounced oscillations take place if $\Omega\nu^2/\nu_0^2\approx 1$. Therefore we could estimate the cutoff frequency for the coherence function (where coherence function approaches zero) as

$$\nu_c^{(m,n)}\approx\nu_0^{(m,n)}/\sqrt{\Omega^{(m,n)}}=U(z)\sqrt{k^{(m,n)}/L\Omega^{(m,n)}}. \quad (14)$$

It is seen that cutoff frequency is a function of current flow spatial distribution $U(z)$, relative frequency spacing $\Omega^{(m,n)}$, and the median sound wave number $k^{(m,n)}$ for different pares of frequencies in the sound signal. For the signal consisting of n different frequencies one can define the number of pares as $N=n(n-1)/2$. In this case we obtain N independent equations (14) for definition of spatial current flow profile $U(z)$. Therefore one could define N independent spatial samplings of $U(z)$, or approximate spatial current flow profile by a polynomial extension of power (*N-1*).

Let's consider an example of current effect on sound spectra variation. Fig.1 shows examples of current profiles, which differ each other only by their maximum positions. Figures 2,3 show the coherence function for phase and amplitude signal variations $\Gamma_{c,S}^{(m,n)}(\nu)$ of log-amplitude fluctuations, given by (13), are plotted as functions of $f=\nu/\nu_0$. It is seen that slow signal variations (f«1) are strongly correlated $\Gamma_{c,S}^{(m,n)}(\nu)\sim 1$. With the fluctuation frequency f increasing, the coherence $\Gamma_{c,S}^{(m,n)}(\nu)$ goes down abruptly and it separates with respect to current profile position on the signal path. Note that the currents symmetrical to the median of the sound path produce the same coherence for the signal variation.

It is possible to give a simple qualitative interpretation to this result be based on the following speculations (see for example [5,9]. It was shown above that the fluctuations with the frequency f appear as a result of interference of waves, which arrive at the observation point with a path difference ΔR_f . The proper signal phase variations will be $k_n\Delta R_f$. Therefore the change of carrier frequency ω_n leads to the shift of this interference pattern, and if this shift turns out to be big enough, the fluctuations on two different carrier frequencies w_m and w_n become non-correlated. To destroy the interference pattern, it is enough to change the wave number k_n on the value Δk, found from the equation $\Delta k \Delta R_f \approx 1$. Mind that $\Delta R_f \approx K^2 L/k^2$, where $K^2 = n^2/U^2$. Therefore $\Delta k \Delta R_f \approx 1$ relation leads to $\Omega f^2 = \Omega(n/n_0)^2 \approx 1$, which in its turn corresponds to the mentioned above results of numerical simulation (see Fig. 2 and Fig. 3). On the other hand, this result means that the amplitude (and phase) fluctuations lose there coherence when the difference of Fresnel zone areas $\Delta S = L/k_m - L/k_n = 2l\Omega / k^{(m,n)}$ for two carrier frequencies w_m and w_n becomes equal to the area of inhomogeneity $K^2 \Delta S \approx 1$.

Thus for the signal consisting of N different pares of we obtain N independent coherence functions for definition of spatial current flow profile $U(z)$. It should note that the method is valid to retrieve $U^2(z)$. Therefore to retrieve the direction of flow and avoid the ambiguity in spatial profile due to symmetrical dependence on distance from the median of the path one should use the additional data on the current.

ACKNOWLEDGMENT

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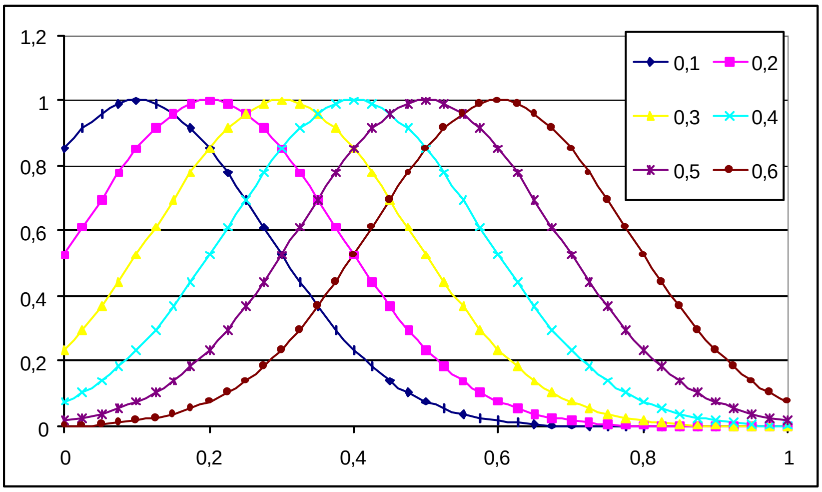

Fig. 1 Flow profiles $U((x-x_0)/L)/U_M$, L is the length of the path. Parameter x_0/L shown at the right top corner

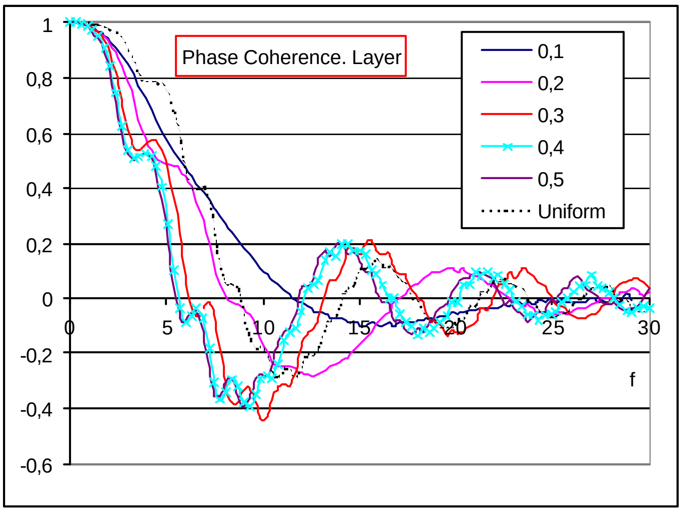

Fig. 2. Phase coherence function, $\Omega = 0.1$, $f = \boldsymbol{n}/\boldsymbol{n}_0$.

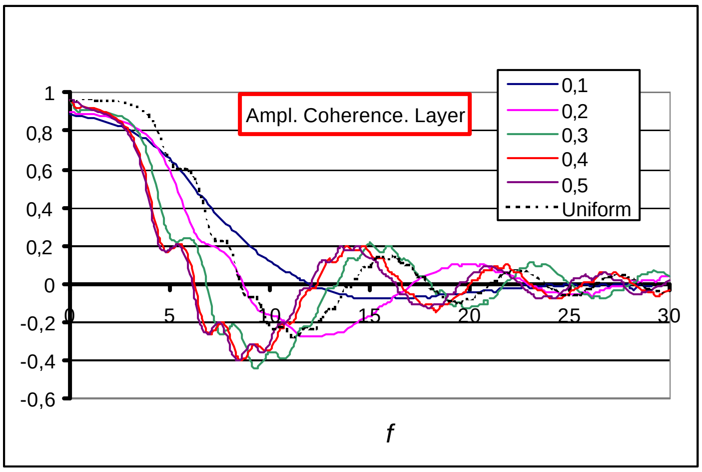

Fig. 3. Amplitude coherence function, $\Omega = 0.1$, $f = \boldsymbol{n}/\boldsymbol{n}_0$.