CONVERGENCE CHARACTERISTICS OF PHASE CONJUGATE WAVES IN SOFAR CHANNELS

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ABSTRACT

To study convergence characteristics of phase conjugate waves in a deep-sea area, we use a SOFAR channel, which is a typical sound speed structure of deep-sea areas. We simulate the sound pressure field and convergence characteristics of the phase conjugate waves in a SOFAR channel by using a normal mode theory based on approximation in which refractive index squared becomes linear. We study how convergence characteristics change with different sound wave frequencies of 50, 100, 150 and 200 Hz. The sound pressure distributions of the phase conjugate waves between a sound source and the transducer array are obtained by changing the range between them from 10 km to 1,000 km. As a result, it is shown that the convergence characteristics of the phase conjugate waves in the SOFAR channel is not affected much by the range between the sound source and transducer array, and good convergence characteristics can be obtained even when the range is several hundred kilometers. It is shown that the sound source and the transducer array are connected by the propagation path similar to the path of eigenray, but they are connected by a bundle-like path rather than a linear path.

1. INTRODUCTION

The phase conjugate wave was first studied in the field of light. Around 1990 the study began in the field of ultrasound as well, and then spread to the field of underwater sound. The study in the field of light has limited applications because it mainly uses nonlinear effects. The frequencies of sound used in the filed of underwater sound, particularly those for oceanographic survey, are mainly low frequencies of several hundred hertz. Low frequency sound wave signal is easy to do signal processing, such as sampling and composition. Therefore it is possible to generate the phase conjugate waves by the signal processing alone without using complicated non-linear effects. Already the experiments for the convergence of the phase conjugate waves in shallow-sea areas have been done successfully 2)3)4)5). The sound waves transmitted from a single sound source placed in a middle depth ware received by a transducer array installed at a range several kilometers away from the sound source. The received signals ware processed by time reversal processors, which ware connected to each transducer, and then re-transmitted from each transducer. Re-transmitted sound waves, namely phase conjugate waves, were confirmed to converge at the position of the sound source that first transmitted the sound waves. This convergence characteristic is not affected by propagation structures between sound source and transducer array, such as sound speed structure and sea floor shape. But it is affected by environmental changes that occur within a time period shorter than the time for a sound wave to make a round trip between the sound source and the transducer array. Therefore it is thought that the convergence characteristics of the phase conjugate waves degrade as the range between the sound source and transducer array increases.

On the other hand, it is known that oceanographic changes are milder in deep seas than in shallow seas. Especially in the oceans where acoustic tomography surveys are carried out, it is observed that temporal changes of ocean structures are mild.

Therefore we assume that oceanographic changes are small, and do simulation by calculation to study convergence characteristics of phase conjugate waves in a SOFAR channel, which is a typical propagation path for sound waves in the ocean.

2. SOUND SOURCE AND ARRAY

Figure 1 shows an arrangement of a sound source and a transducer array. Sound wave transmitted from the sound source placed at point $\mathbf{r}_s$ is received by the transducer array placed at point $\mathbf{r}_n$. After the phase conjugate process by each transducer, the phase conjugate waves are transmitted from each transducer. The phase conjugate wave formed

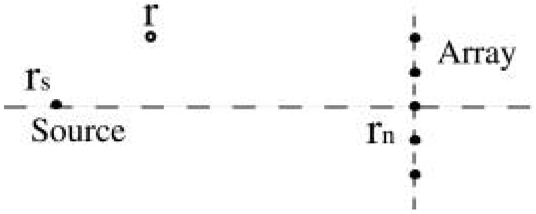


Fig.1 Geometry of sound source and transducer array.

at point **r** is expressed by the following Green function [1)]:

$$G_{c\,\omega}(\mathbf{r}, \mathbf{r}_s) = \sum_{n=1}^{N} \boldsymbol{G}^{*}_{\omega}(\mathbf{r}_n, \mathbf{r}_s)\, \boldsymbol{G}_{\omega}(\mathbf{r}, \mathbf{r}_n) \quad (1)$$

where $\mathbf{G}_{,}(\mathbf{r}, \mathbf{r}_s)$ is a Green function to describe the sound field at point $\mathbf{r}_n$ of transducer array that is generated by the sound source placed at point $\mathbf{r}_s$, and $\mathbf{G}_{,}(\mathbf{r}, \mathbf{r}_n)$ is a Green function to describe the sound field generated at point **r** by the transducer array placed at point $\mathbf{r}_n$. Asterisk * denotes phase conjugate. The array is assumed to consist of N pieces of single non-directional elements.

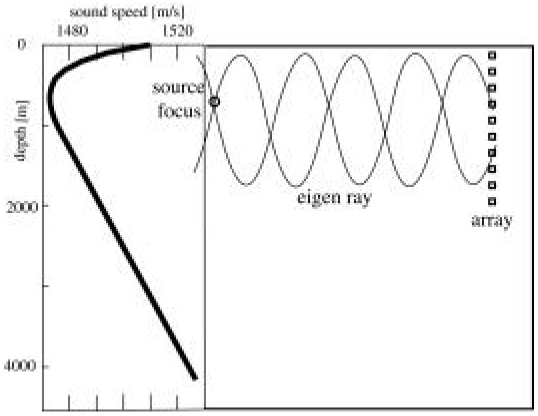


Fig.2. Sound speed profile and geometry of source and array.

3. MODEL OF SOUND WAVE PROPAGATION IN DEEP SEA

To study the effects of the phase conjugate waves in deep seas, we first compare the behavior of the phase conjugate waves with that of ordinal propagation waves. The propagation model for calculation of sound field in deep sea is the normal mode theory in which refractive index squared is approximated as linear. Figure 2 shows the sound speed profile used for the simulation. This profile is divided into 11 layers in which refractive index squared becomes linear. The sound pressure in each layer is expressed by the following equation [7)]:

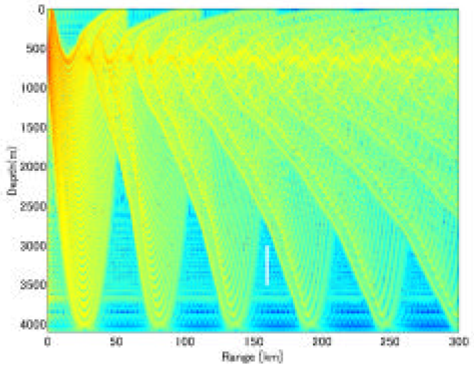


Fig.3. Sound pressure distribution of normal propagating waves in SOFAR channel.

$$p(z, k) = A_n Ai[Z_n(z)] + B_n Bi[Z_n(z)], \qquad (2)$$

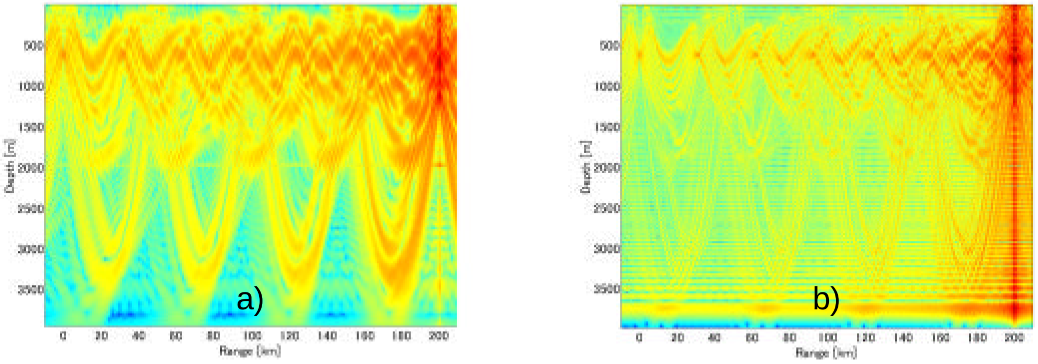
where *Ai* and *Bi* are Airy functions, $Z_n(z)$ is normalized depth, and k is a wave number. By applying boundary conditions of sea surface and sea floor to this equation, the sound pressure at the point (r,z) is expressed by the following equation:

$$p(r,z,z_0) = \frac{i}{2\boldsymbol{r}_0} \times \sum_{m=1}^{M} C_0(k_m)\, p(z_0,k_m)\, p(z,k_m) H_0^{(1)}(k_m r)\ , \qquad (3)$$

where

$$C_0(k_m) = k_m[H_m p^2(z_N,k_m) + \boldsymbol{w}^2 \boldsymbol{b}_N L_N^2 \pi^{-1} \boldsymbol{r}_N^{-1}(A_N B_N - B_N \dot{A}_N)]^{-1}, \ (4)$$

and z_0 is the depth of the sound source, k_m is the wave number of m-th mode. Other variables can be seen in References [9]. By substituting the Green function based on equation (3) into equation (1), we obtain an equation to express the sound pressure of the phase conjugate waves at any point.



3. NUMERICAL RESULTS

We first obtain a sound pressure distribution of ordinary propagation waves in the deep sea that has the profile shown in Figure 2. Figure 3 shows the sound pressure distribution of ordinary propagation waves transmitted from the sound source of frequency 200 Hz placed on the SOFAR axis (depth 630m). It can be seen that the sound waves spread vertically from the sea surface to the depth of 4,000 meters. However, there are periodical ranges where the vertical width of the sound field becomes narrow. At these positions, we can sample the sound field with a short array of transducers. In the following calculation, we consider a transducer array that is installed vertically from the sea surface to the depth of 1,200 meters. The intervals between transducer array elements are equal to the wavelength.

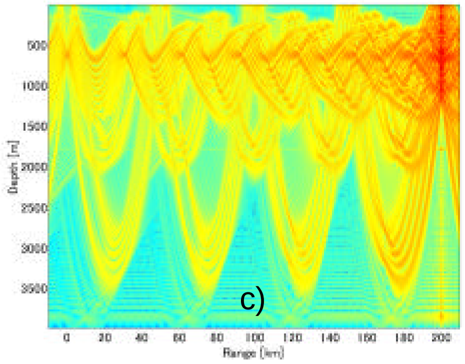


Fig.4. Sound pressure distribution of phase conjugate waves.
Frequency; a): 50 Hz, b):100 Hz, c):150 Hz

To apply the phase conjugate waves in the sea to oceanographic measuring, such as the ocean acoustic tomography, it is necessary to know the propagation path between the sound source and transducer array. First we study how convergence characteristics of the phase conjugate waves change with frequency of sound wave. Figure 4a),b),c) show the sound wave distributions of the phase conjugate waves when the frequency is 50, 100, and 150 Hz, respectively, where the range between the sound source and transducer array is 200 km. For all these frequencies, it is seen that a clear focus is formed at the position of the sound source (range 0m, depth 630m). When we compare this figure with the convergence pattern in a shallow sea, we can see that this figure is sharper and smoother than those of the shallow sea $^{6)}$. The reason for this may be that the number of modes occurred in deep sea is about 10 times as that in shallow sea.

Next we set the frequency of sound waves at 200 Hz, and change the range between the sound source and transducer array to study the sound pressure between them and convergence characteristics around the sound source.

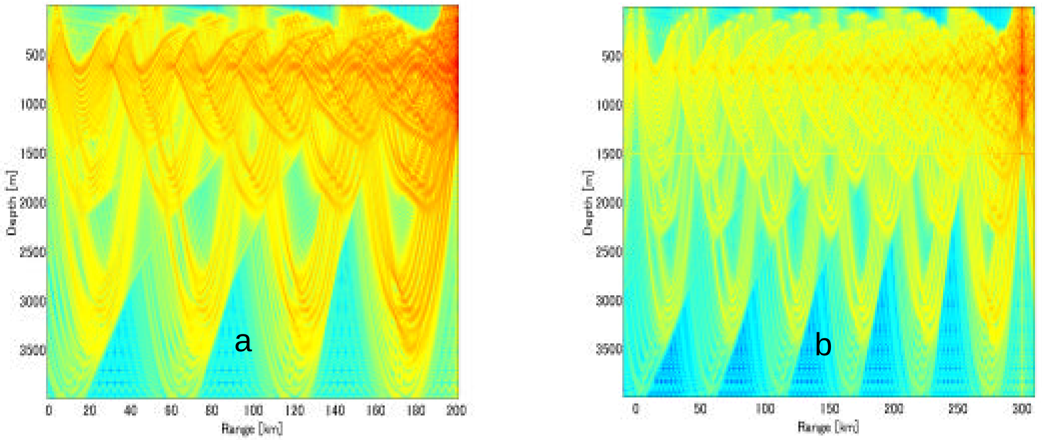


Fig.5. Sound pressure distribution of phase conjugate waves.

Range; a): 200 km, b):300 km

Figure 5a) shows the sound pressure distributions of the phase conjugate waves obtained when the range between the sound source and transducer array is 200 km. It is seen that all the waves converge to the position of the sound source (the point of range 0 , depth 630 m) and that the paths from the transducer array to the sound source is largely divided into three. Figure 5b) shows the sound pressure distribution of the phase conjugate waves when the range is 300 km. It is seen that all the waves converge to the position of sound source. But the number of paths connecting from the transducer array to the sound source increases to 4 and the cycle numbers of the paths also increase. These paths are similar to the eigenray connecting the two points used for the ocean acoustic tomography, namely the sound source and the hydrophone. However, in this case where an array of transducers is used instead of a single transducer, the paths are not formed as a line but is formed as a bundle. Therefore the area passed by phase conjugate waves is wider than the area passed by eigenray of ordinary waves.

Figure 9 shows magnified graphs of sound pressure distributions around the sound source when the distance is 200km and 1,000km, respectively. It appears that effect by distance is small. Figure 10 show vertical distributions around the sound source when distance is taken as a parameter. For some distances, sampling of sound field is not enough, but there seems little effect on convergence characteristics. It is clear that a focus of phase conjugate waves is formed even for such long distances.

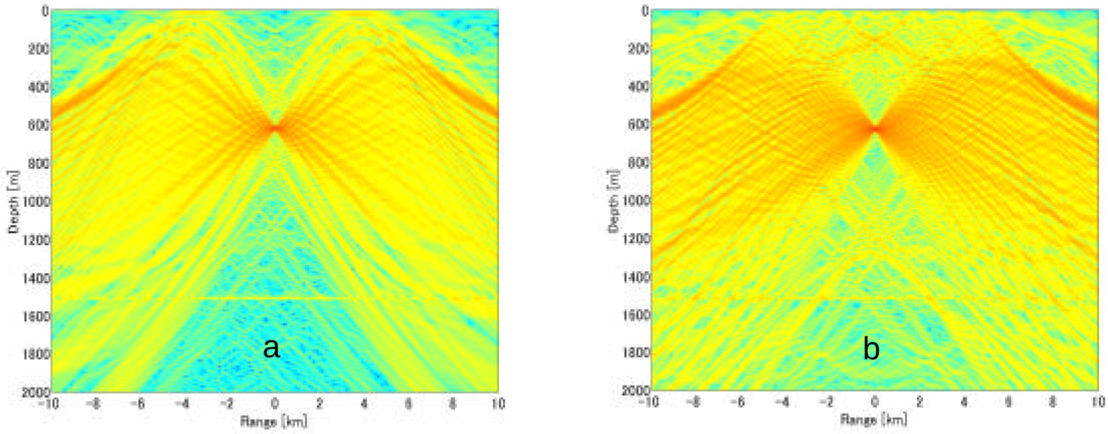


Fig. 6. Sound pressure distribution of phase conjugate waves near focus. a; 300 km, b; 1000 km

4. SUMMARY

We studied the convergence characteristics of the phase conjugate waves in deep sea by using a typical SOFAR channel. We used the normal mode theory for sound field analysis. For a range of frequencies from 50 Hz to 200 Hz, the clear convergence to the position of the sound source was obtained. The paths from the sound source to transducer array are formed the bundle-like path similar to the eigenray in the ray theory. It was also made clear that the phase conjugate waves in SOFAR channel converge to the sound source even when the propagation range is as long as 1,000 km. This tendency is clearer for the vertical distribution than for the horizontal distribution. It was generally confirmed that the phase conjugate waves in deep sea are not much affected by range.

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