PSYCHOACOUSTIC OPTIMIZATION OF GENERATED AUDIO TEST SIGNALS

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ABSTRACT

There are several types of test signals that can be used for measurement of transfer functions of electroacoustic systems, harmonic signals with a fixed frequency, sweeping signals, complex signals consisting of several waveforms or random signals. This contribution deals with design and digital generation of a complex signal consisting of a sawtooth test signal that passes through all the levels of *A/D* converter plus an additional low-level harmonic signal. During generation of signals, additional unwanted frequency components are produced, dependent on both the sampling frequency and signal amplitude. In order to minimize the impact of these newly created undesirable distorting components on audible perception, a dither is added to the signal. For test signals designated for audible tests, a dither with triangular probability density function was used.

COMPLEX TEST SIGNALS

For testing of properties of electroacoustic systems, linearity of *A/D* and *D/A* converters, both by measurement and by listening, it is possible to use a test signal comprising of two components. One component is a sawtooth signal $x_t(t)$, the amplitude of which corresponds to the range of digital conversion, which is shown in Fig.1a. A detail Fig.1b shows this to be actually a staircase function. Discrete values of this signal correspond to appropriate quantizing levels of a 16-bit converter, used for recording and reproduction of the signal on a *CD*. The second component is formed by a harmonic signal $x_h(t)$ of a frequency within a spectrum of maximum sensitivity of human ear, that is in the 1 to 4 *kHz* range [1]. The amplitude of this harmonic signal is relatively low, in the order of several quantizing steps of the converter.

The complete test signal $x_r(t)$ is designed in such a way that on each level of the sawtooth signal there is superimposed exactly one period of harmonic signal having a length of ΔT_1. Fig. 2 shows a detail of this complex test signal. The amplitude range of the test signal $x_r(t)$ is $\pm 1\,V$.

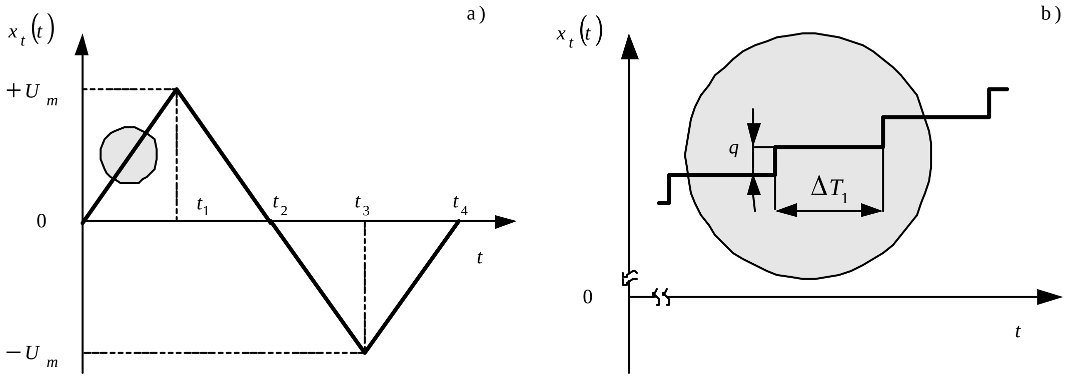

Fig. 1. Staircase sawtooth signal $x_t(t)$; a) total signal range; b) rising part of the signal detail.

Analytical expression for the sawtooth signal $x_t(t)$ can be assembled in such a way that we use a sum of shifted step functions $1(t)$, having a magnitude equal to one quantizing level of the converter q. The shift of functions $1(t)$ equals to the length of one period of the added low-level harmonic signal $x_h(t)$. The sawtooth signal $x_t(t)$ can the be expressed as

$$
\begin{aligned}
x_t(t) = \; & q \sum_{k=1}^{\infty} 1(t - k\Delta T_1) && \text{........ rising signal component } 0 \le t \le t_1 \text{ ,} \\
& -2q \sum_{l=1}^{\infty} 1(t - l\Delta T_1 - t_1) && \text{........ falling signal component } t_1 < t \le t_3 \text{ ,} \\
& +2q \sum_{m=1}^{\infty} 1(t - m\Delta T_1 - t_3) && \text{........ rising signal component } t_3 < t \le t_4 \text{ ,} \\
& -q \sum_{n=1}^{\infty} 1(t - n\Delta T_1 - t_4), && \text{........ zero level at the signal end } t > t_4 \text{ ,}
\end{aligned}
\tag{1}
$$

where time constants t_1, t_3 and t_4 determine the point in time where the sawtooth signal changes, as can be seen in Fig. 1a.

The complete test signal $x_r(t)$ will be created as a sum of the sawtooth signal $x_t(t)$ and the low-level harmonic signal $x_h(t)$

$$x_r(t) = x_t(t) + x_h(t). \tag{2}$$

Signals $x_t(t)$, $x_h(t)$ and $x_r(t)$ and their mutual phase relationships are shown in Fig. 2.

SIGNAL ANALYSIS

Test signal $x_r(t)$ crosses all the converter levels by the defined way. Quality of an electroacoustic system can be judged either by digital measurement with subsequent output signal analysis, or by listening test. Frequency spectrum of a test signal $x_r(t)$ can be obtained by Fourier transformation (*FT*) and from (2) it can be written in the form

$$X_r(w) = FT\{x_r(t)\} = FT\{x_t(t) + x_h(t)\} = FT\{x_t(t)\} + FT\{x_h(t)\}. \tag{3}$$

A resulting spectrum of signal $x_r(t)$ can be obtained by frequency analysis of its components.

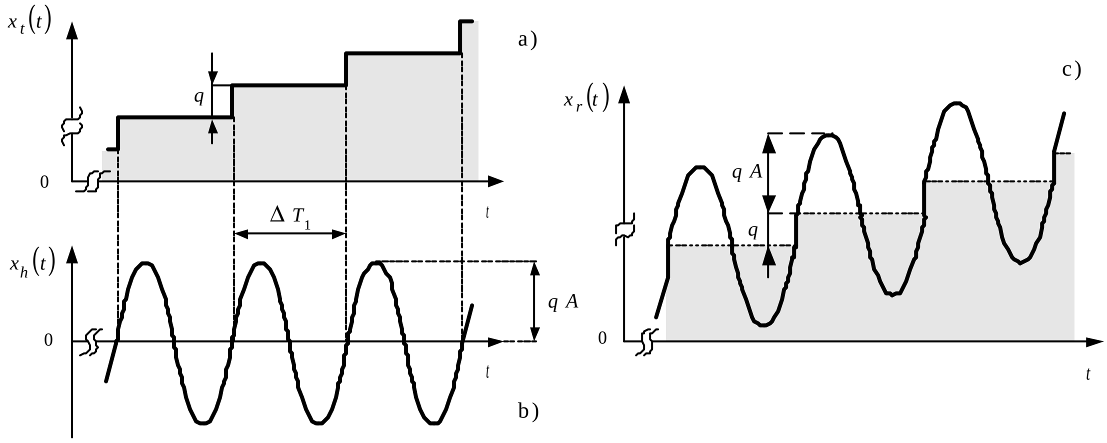

Fig. 2. A portion of total signal $x_r(t)$; a) detail of rising part of a sawtooth signal $x_t(t)$; b) low level harmonic signal $x_h(t)$; c) sum of signals $x_t(t)$ and $x_h(t)$.

If we consider, during design of a test signal $x_r(t)$, a harmonic signal $x_h(t)$ having frequency 1470 *Hz*, then its corresponding wavelength is $\Delta T_1 = 0{,}68 \ ms$. Harmonic signal has a period of exactly 30 samples, with sampling frequency of 44.1 kHz, which is used in *CD* recording. This will ensure that on every quantizing level there will be just one whole period of harmonic signal $x_h(t)$.

The overall length of a test signal T_r can be determined from formula (1). From the character of designed test signal follows that during reproduction stage the sawtooth signal runs twice through the converter's whole range. Assuming a 16-bit conversion, the total number of steps is $2^{(16+1)}$. As long as the time length of each step is equal to one period of the superimposed harmonic signal ΔT_1, we can simplify the formula for the signal length $T_r = 2^{(16+1)} \Delta T_1 = 89{,}16 \ s$. As can be seen from Fig. 1, the rising part of sawtooth signal $x_t(t)$, defined within a time frame $0 \le t \le t_1$ lasts for $t_1 = T_r / 4 = 22{,}29 \ s$. This time is sufficiently long to form a stable impression of the reproduced test signal from a psychoacoustical point of view, regardless of whichever part of the signal $x_r(t)$ will be reproduced next, either the one in the interval $0 \le t \le t_1$ and $t_3 \le t \le t_4$, or the falling part in the interval $t_1 \le t \le t_3$ [1]. For this reason we can limit our attention, while analyzing the sawtooth signal $x_t(t)$, on one of its parts, for instance its initial rising part in $0 \le t \le t_1$, which we shall mark as $x_{t1}(t)$.

From the mathematical analysis point of view, it will be better to extend the selected signal part $x_{t1}(t)$ theoretically up to a time unlimited range, so that we can write

$$x_{t1}(t) = q \sum_{k=-\infty}^{\infty} 1\left(t - k\,\Delta T_1\right). \qquad (4)$$

For a frequency analysis of sum of signals shifted in time domain (4) we can use a formula, valid for the *Fourier* transformation [2, 3]

$$FT\left\{\sum_{n=-\infty}^{\infty} f(t - nT_0)\right\} = \frac{2p}{T_0} \sum_{k=-\infty}^{\infty} F(w)\, d(w - k\frac{2p}{T_0}) \,, \qquad (5)$$

where $f(t)$ is a signal in time domain and $F(w)$ is its corresponding picture in frequency domain, T_o is the time shift.

Substituting (4) into (5) we obtain

$$FT\left\{q\sum_{n=-\infty}^{\infty}1(t-n\Delta T_1)\right\}=\frac{2p\,q}{\Delta T_1}\sum_{k=-\infty}^{\infty}FT\ \{1(t)\}d\,(w-k\frac{2p}{\Delta T_1})\ , \qquad (6)$$

Spectrum of a unity step function $1(t)$ is determined by a formula [2]

$$FT\ \{1(t)\}=p\ d\ (w)-\frac{j}{w}\ . \qquad (7)$$

Frequency spectrum $X_{t1}(w)$ of the signal $x_{t1}(t)$ can be expressed on the basis of (6) and (7) as

$$X_{t1}(w)=\frac{2p\ q}{\Delta T_1}\sum_{k=-\infty}^{\infty}FT\ \{1(t)\}d\left(w-k\frac{2p}{\Delta T_1}\right),$$

$$X_{t1}(w)=\frac{2p\ q}{\Delta T_1}\sum_{k=-\infty}^{\infty}[p\ d\ (w)-\frac{j}{w}]d\left(w-k\frac{2p}{\Delta T_1}\right)=R_e(w)+jI_m(w). \qquad (8)$$

Signal spectrum (8) contains a real $R_e(w)$ and an imaginary $I_m(w)$ component

$$R_e(w)=\frac{2p\ q}{\Delta T_1}\sum_{k=-\infty}^{\infty}p\ d(w)\ d\left(w-k\frac{2p}{\Delta T_1}\right)=\frac{2p^2 q}{\Delta T_1}d(w)\ ,$$

$$I_m(w)=-\frac{2p\ q}{\Delta T_1}\sum_{k=-\infty}^{\infty}\frac{1}{w}d\left(w-k\frac{2p}{\Delta T_1}\right). \qquad (9)$$

A real component $R_e(w)$ represents only a *DC* signal component, which is not particularly interesting from the frequency analysis point of view. The imaginary component $I_m(w)$ represents discrete frequencies with a $2p/\Delta T_1$ step.

DSP OF TEST SIGNAL

So far, we considered a continuous test signal $x_r(t)$, which represents analog signal in reality. In actual realization, however, the test signal will be recorded on *CD*, meaning the theoretically designed signal will be subjected to *DSP* process. Due to the fact that the stepped sawtooth signal $x_t(t)$ had been designed so that its individual steps are exactly the quantizing steps, no quantizing error takes place and therefore there is no quantizing noise.
Harmonic signal $x_h(t)$ can be described as

$$x_h(t)=A\ q\sin\ (2p\ \frac{1}{\Delta T_1}t)\ , \qquad (10)$$

where A is an integer number of quantizing levels q. Quantizing of a harmonic component of the test signal $x_h(t)$ will result in appearance of new harmonic components [3, 4].

The overall expression for quantizing harmonic signal $x_h(t)$ may be, according to [5] written in the following form

$$x_g(t) = A\,q\sin\left(\frac{2p\,t}{\Delta T_1}\right) + \frac{2\,q}{p}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sum_{m=1}^{\infty} J_m(2p\,n\,A)\sin\left(\frac{2p\,m\,t}{\Delta T_1}\right), \qquad for \quad m \quad odd\;, \tag{11}$$

where $J_m(.)$ is a *Bessel* function of the first kind of order m. Equation (11) contains the fundamental component $A\,q\,\sin\left(2p\,t/\Delta T_1\right)$ and odd error harmonic components.

A sound signal is recorded on *CD* with a sampling frequency $f_s = 1/\Delta T_s = 44.1\ kHz$, where ΔT_s is the sampling step. Discretization of the signal $x_q(t) = x_t(t) + x_g(t)$ can be described in the time domain

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_q(t)\,d\,(t - n\,\Delta T_s) = \sum_{n=-\infty}^{\infty}\left[x_t(t) + x_g(t)\right]d\,(t - n\,\Delta T_s), \tag{12}$$

Whereas in the frequency domain the following applies

$$X_s(w) = FT\,\{x_s(t)\} = \frac{1}{\Delta T_s}\sum_{k=-\infty}^{\infty} X_r\left(w - k\frac{2p}{\Delta T_s}\right). \tag{13}$$

Signal discretization will manifest itself as a presence of aliasing in the resulting test signal spectrum $X_s(w)$.

PSYCHOACOUSTIC MODIFICATION OF TEST SIGNALS

For testing of electroacoustic systems by measurement, a suitable signal is the designed digital signal (12). Knowing the input signal at the device under test as well as a digitally processed response, we can carry out a detailed analysis of transfer properties of the tested system. When the test signal is to be used for psychoacoustic testing, it is desirable to minimize the impact of distorting components (11) and (13) on its perception. There are several methods of *DSP*. These include a non-linear quantizing of signals, noise shaping and dithering [4, 6, 7].

For generation of low-level test signals for listening tests a non-subtractive dithering was used. The simplest type of signal that can be used as dither is a noise with rectangular probability density function. A more appropriate type of dither is a signal $J(t)$ with a triangular probability density function, which can be written in the following form

$$p_J(J) = \begin{cases} \dfrac{q - |J|}{q^2}, & -q \le J \le q, \\ 0\;, & otherwise\;. \end{cases} \tag{14}$$

The low-level signal with dither, despite a higher noise level, is perceived as acoustically clear.

CONCLUSIONS

A test signal, recorded on a *CD* can be generally used for evaluation of transfer functions of electroacoustic systems by both measurement and listening [8, 9]. It is also suitable for verification of digital transfer between individual studio components. Responses of systems to the test signal are recorded and analyzed in both time and frequency domain.

Fig. 3 shows analysis of a signal without dither recorded on a *CD*, played back on a *CD* player. Reproduced signal, displayed in a time domain in Fig. 3a, shows errors that are clearly identifiable both by digital analysis and during listening tests. Fig. 3b shows a measured spectrogram in the vicinity of errors in a reproduced stereophonic signal that took place in both left and right channel. In the time domain, the corresponding interval is shown in Fig. 3d.

Fig 3c. shows a detail of an error free signal reproduction. During this error free playback, we can hear the original harmonic signal, together with additional spectral components that appeared due to signal discretization and quantizing.

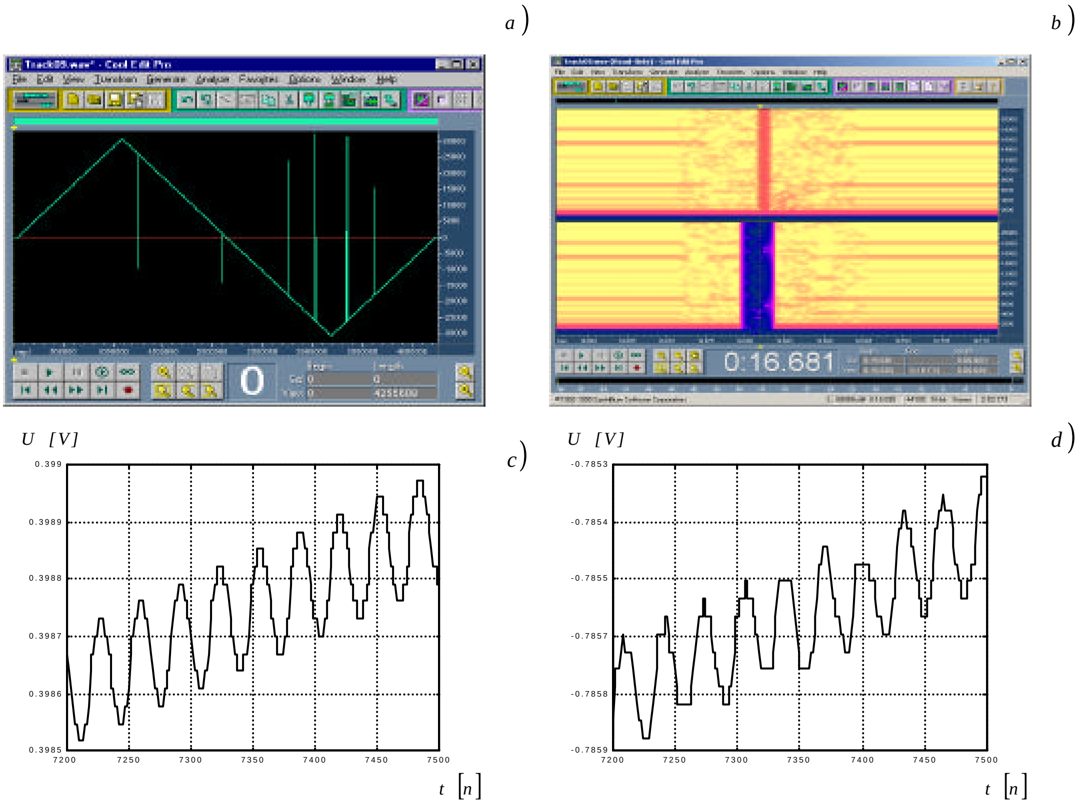

Fig. 3. Analysis of a recorded and reproduced test signal $x_s[n]$.

As long as the system is without problems, the reproduced signal will have a consistent character, without any additional sound effects. However, if during the passage of the test signal through the system some errors take place, such as in Fig. 3a and Fig. 3b there are dropouts of the harmonic tone and an additional hiss, swishing sound and crackling are heard. Various errors, created during digital signal processing, are easy to detect either by measurement or by listening.

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