Reflection waves by a submerged layer with density sound velocity and attenuation symmetrical gradients

PACS REFERENCE: 43.20.Gp

Carbó, Rafael (1) ; Molero, Adriana C. (2)
(1) Instituto de Acústica, CSIC
Serrano 144,
28006 Madrid
Spain
915618806
914117651
e-mail: iaccf31@ia.cetef.csic.es
(2) Comprehensive Nuclear-Test-Ban Treaty OrganizationVienna International Centre.
P.O. Box 1200A-1400
Vienna
e-mail:Adriana.Molero.Alonso@ctbto.org

ABSTRACT

Special composite materials could be modeled as an inhomogeneous medium where the density, sound velocity and attenuation of acoustical energy varies. This paper presents a theoretical study of sound wave propagation through media whose properties change along one of the coordinate rectangular axis, properties that remain constant in the plane orthogonal to this axis. The results were applied to obtain the reflection loss of a symmetrical inhomogeneous layer with a central homogeneous layer of thickness D and acoustic impedance $\rho\ c$, adapted to the water impedance $\rho_0\ c_0$ in both surfaces by a viscoelastic layer of thickness d, with doping of load particles with given density sound velocity and linear gradient attenuation. We assumed the following conditions: low particles concentration, low particle sizes as compared to the wavelength, normal incidence of the acoustic waves and thickness D>>d

1. INTRODUCTION

Wave reflection from an inhomogeneous layer of the simplest form was studied by Brekhovskikh [1] in the cases of transitional and symmetrical gradients of density and sound velocity. In some previous papers [2,3] we have deal with the reflection coefficient from a transitional layer with gradients of density sound velocity and absorption applied to the case of turbid waters and unconsolidated sedimentary layers.

Robins [4] derives analytical solutions of the Helmoholtz equation for the reflection of plane acoustic waves from a layer of varying density between two homogeneous media. In another paper Robins [5] considers the transmission of an acoustic plane wave through a horizontally stratified fluid layer whose density and sound speed both vary continuously with depth. Other authors [6,7] study the reflection coefficient of a layered elastic seabed with progressive adaptation of acoustic impedance.

A review of plane wave propagation through a layer with density and sound velocity gradient is given, and we introduce the effect of a sound absorption energy gradient when the layer is submerged into the water. In this way the Urick [8] suspension theory is considered.

2. THE GRADIENT MEDIUM

In a gradient medium where the number of included particles n(x) increases through the x axis, the density is written as

$$\rho(x)= \rho_1 \beta(x)+\rho_2[1-\beta(x)] \quad , \qquad \beta(x)=n(x)V_p/V \tag{1}$$

where ρ_1 is the density of the particles, V_p the volume of one particle, V the total volume, and ρ_2 the density of suspended material.

A model of sound propagation in a suspension based on scattering theory was first formulated by Urick [8]. Considering the scattered waves from a number of randomly spaced particles, the sound propagation through the composite material is characterized by the sound wave velocity and the absorption coefficient. At low frequencies the sound velocity could be expressed by the Wood equation [9] applied to a suspension (such as mineral particles in water), or to any medium lacking rigidity:

$$c(x) = 1/[\rho(x)\chi(x)]^{1/2} \tag{2}$$

where χ is the total compressibility expressed by:

$$\chi(x) = \beta(x)\chi_1 + [1-\beta(x)]\chi_2 \tag{3}$$

χ_1 is the compressibility of the particles and χ_2 the compressibility of the agglomerate. At high frequency Hoven [10] predicts an increased velocity caused by the decrease in the effective density due to relative particles-suspending material movement.

The absorption of sound in a suspension of particles was deduced by Urick, provided the size of the particles a, is small compared to the wavelengt λ. The expression of the absorption coefficient α, if there are n(x) particles per unit volume is

$$2\alpha = n(x) V_p k \{ (k^3a^3/3) + \sigma(\mu-1)^2/[\sigma^2+(\mu+\nu)^2]\} \tag{4}$$

where $k=2\pi/\lambda$ is the wavenumber and

$$\mu=\rho_1/\rho_2 \ ; \quad \nu=0.5+(9/4\xi a) \ ; \quad \sigma=(9/4\xi a)[1+(1/\xi a)] \ ; \quad \xi=(\omega\rho_2/2\eta)^{1/2} \tag{5}$$

The first of the two terms on the right of the equation (4) is the scattering loss produced by small rigid spherical particles free to move in the sound field, and represents a redistribution, rather than a dissipation, of the energy. The second term is a frictional loss due to the viscosity η of the suspending material. When the particles are suspended into a solid medium the viscosity becomes zero, and the absorption coefficient remains only dependent on the scattering losses.

This expression of the absorption coefficient shows a linearity with the number of particles valid only at low concentration where particle interactions can be neglected. High concentrations need a more complicate deduction, like the Biot theory [10] for a porous medium.

At low frequency the limit expressions for the viscous attenuation in both Biot theory and suspension theory are very similar, having the same dependence on frequency and density ratio. The difference between the two expressions is in the relationship between particle size, porosity, and permeability. At low frequencies the suspension model gives the same attenuation as the Biot model if the latter uses adequate values of permeability. In the extreme high frequency region, normally the Biot attenuation will be somewhat higher than the suspension model since the implied permeability in the suspension model is higher. On the other hand the suspension model give much too high attenuation values for concentration above a few percent, the Biot model gives the pleasingly correct results of zero viscous attenuation at 100% concentration.

3. ACOUSTIC FIELD IN AN INHOMOGENEOUS MEDIUM.

The fundamental equations of the acoustic field in an inhomogeneous medium have the form [11]

$$\delta p / \delta t = - \rho c^2 \text{ div } \mathbf{v}$$

$$\delta \mathbf{v} / \delta t = - 1/\rho \text{ grad } p \qquad (6)$$

where p is the acoustic pressure, $\mathbf{v}$ is the particle velocity in the wave and c the velocity of sound propagation. In the general case ρ and c are functions of position. The first of these equations is the equation of continuity and the second the Euler equation. Assuming harmonic time dependence, $\delta/\delta t = -i\,\omega$, and eliminating $\mathbf{v}$ from equations (6), we obtain

$$\rho \text{ div } [(1/\rho) \text{ grad } p] + k^2 p = 0 \qquad (7)$$

$$\nabla^2 p + k^2 p - (1/\rho) \text{ grad } \rho \text{ grad } p = 0 \qquad (8)$$

Introducing in place of p a new potential ϕ, defined by

$$\phi = p / (\rho^{1/2}) \qquad (9)$$

after some transformations, we obtain the wave equation for ϕ

$$\nabla^2 \phi + \mathbf{K}(x)^2 \phi = 0 \qquad (10)$$

where

$$\mathbf{K}(x)^2 = \mathbf{k}(x)^2 + (1/2 \rho(x)) \nabla^2 \rho(x) - (3/4) [1/\rho(x)) \text{ grad } \rho(x)]^2 \qquad (11)$$

This wave equation can be integrated in several cases, for example when the density varies according to a exponential law and the complex wave number remains constant, or when the velocity of sound propagation varies according to

$$c(x) = c_1 / (1+\gamma x)^{1/2}$$

But a general solution of the wave equation (10) is not possible, nevertheless we solve the problem by imagining a layer consisting of a large number of thin, plane parallel, homogeneous layers in contact with one another . In passing from layer to layer, the properties of the medium change discontinuously. However, letting the thickness of the layers approach zero while their number approaches infinity, we obtain a layered-inhomogeneous medium with continuously varying parameters.

4. REFLECTION BY AN INHOMOGENEOUS LAYER SUBMERGED IN WATER.

Let us consider a one-dimensional inhomogeneous layer in the x-axis direction, with a thickness d, submerged in an homogeneous medium (water) with acoustic impedance $\rho_0 c_0$. From this medium a plane wave sound beam at normal incidence strikes the boundary surface of the layer (see figure 1).

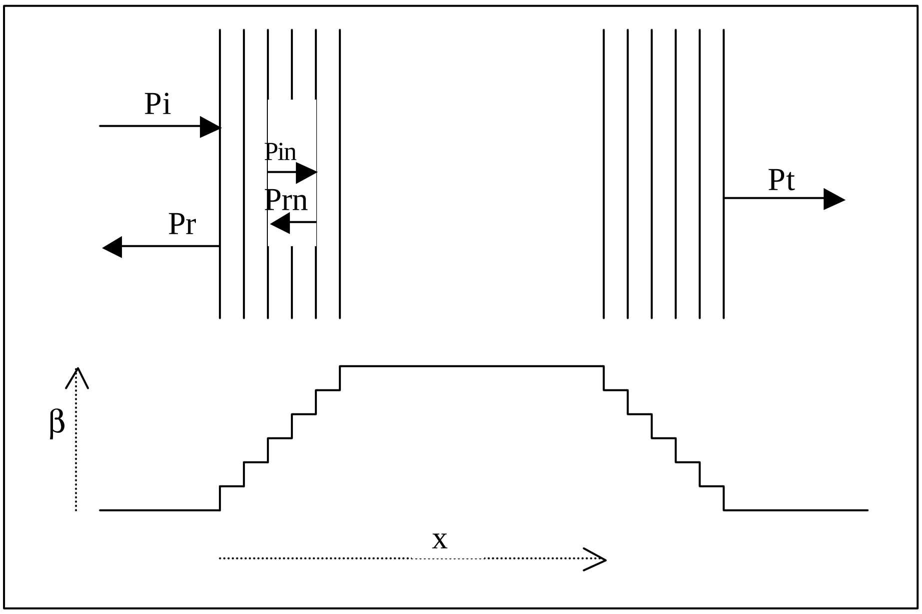

Fig. 1. Schematic description of the symmetrical layer

Now we can get the pressure of the incident and reflected waves in the water

$$\mathbf{P_i} = \mathbf{A_0} \exp[\, i(\omega t - k_0 x)] \quad , \qquad \mathbf{P_r} = \mathbf{B_0} \exp[\, i(\omega t + k_0 x)] \qquad (12)$$

and the velocity of the particles

$$\mathbf{v_i} = (1/z_0)\, \mathbf{A_0} \exp[\, i(\omega t - k_0 x)] \quad , \qquad \mathbf{v_r} = (-1/z_0)\, \mathbf{B_0} \exp[\, i(\omega t + k_0 x)] \qquad (13)$$

where $z_0 = \rho_0\, c_0$ is the acoustic impedance, k the wavenumber in the water and $\mathbf{A_0}$, $\mathbf{B_0}$ the complex amplitudes of these waves.

Assuming that a plane wave is propagated through the inhomogeneous layer divided into m homogeneous layers in contact, the pressure and the particle velocity in the n layers will be

$$\mathbf{P_{in}} = \mathbf{A_n} \exp[\, i(\omega t - \mathbf{k_n} x)] \quad , \qquad \mathbf{P_{rn}} = \mathbf{B_n} \exp[\, i(\omega t + \mathbf{k_n} x)] \qquad (14)$$

$$\mathbf{v_{in}} = (1/\mathbf{z_n})\, \mathbf{A_n} \exp[\, i(\omega t - \mathbf{k_n} x)] \quad , \qquad \mathbf{v_{rn}} = (-1/\mathbf{z_n})\, \mathbf{B_n} \exp[\, i(\omega t + \mathbf{k_n} x)] \qquad (15)$$

where $\mathbf{z_n} = \rho(x_n) c(x_n)/(1 + i\alpha_n/k_n)$ is the complex impedance of the layer n and $\mathbf{k_n} = k_n + i\alpha_n$ the complex wavenumber, where the imaginary part is the absorption coefficient of the nth layer. Since the absorption coefficient is dependent on the frequency, the complex impedance is also dependent of the frequency.

Finally, in the right half space of water only a transmitted wave travels in the positive x axis direction, with pressure and particle velocity

$$\mathbf{P_t} = \mathbf{A_t} \exp[\, i(\omega t - k_0 x)] \quad , \qquad \mathbf{v_t} = (1/z_0)\, \mathbf{A_t} \exp[\, i(\omega t + k_0 x)] \qquad (16)$$

There are two boundary conditions that must be satisfied for all times at all points on the m+1 boundaries between the layers: the acoustic pressures and the particle velocities on both sides of each boundary surface are equal. Applying this conditions we obtain a pair of equations at each n boundary given by the matrix expression

$$\begin{bmatrix} A_n \\ B_n \end{bmatrix} = \mathbf{T_n} \begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} \qquad (17)$$

where $\mathbf{T_n}$ is the matrix

$$\mathbf{T_n} = \begin{bmatrix} (z_{n+1}+z_n)\exp[-i(k_{n+1}-k_n)x_n] & (z_{n+1}-z_n)\exp[i(k_{n+1}+k_n)x_n] \\ (z_{n+1}-z_n)\exp[-i(k_{n+1}+k_n)x_n] & (z_{n+1}+z_n)\exp[i(k_{n+1}-k_n)x_n] \end{bmatrix} \qquad (18)$$

By applying m times the equation (17) we obtain the relation between the amplitude of the pressure on both sides of the inhomogeneous layer

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \mathbf{T_0}\ \mathbf{T_1}\ \ldots\ldots\ \mathbf{T_{m-1}} \begin{bmatrix} A_t \\ 0 \end{bmatrix} \qquad (19)$$

The complex reflection coefficient of the inhomogeneous layer will be determined by

$$\mathbf{R} = \mathbf{A_0}/\mathbf{B_0} \qquad (20)$$

Now we apply this approximate method to some examples considered below in order to calculate the reflectivity frequency response and the shape of the reflected pulse by the inhomogeneous symmetrical layer submerged in water.

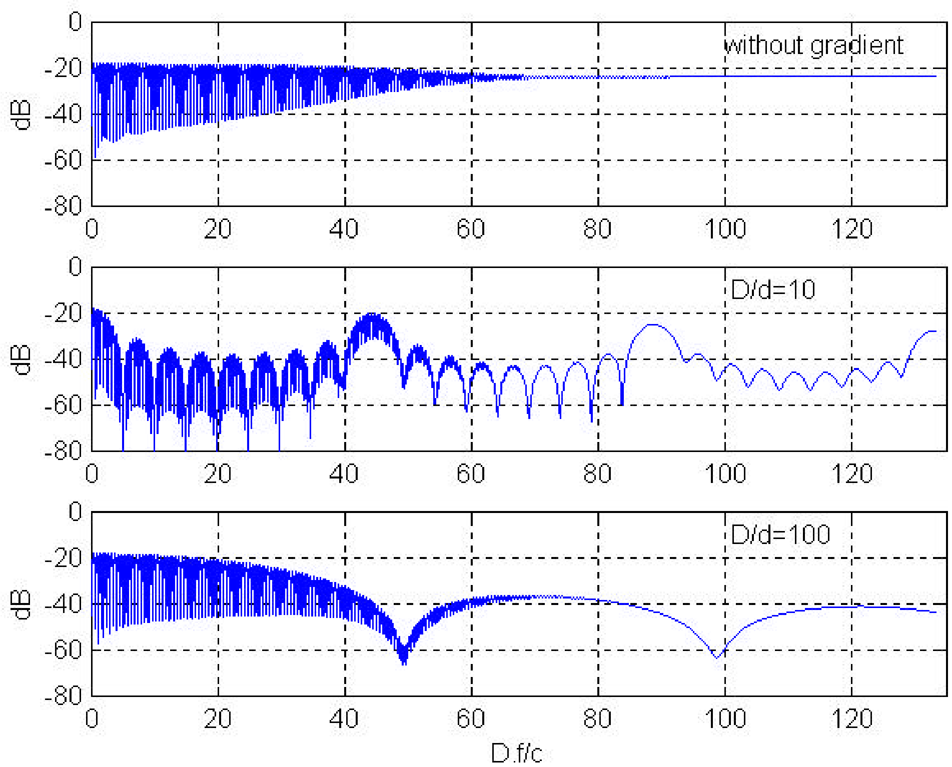

Fig.2. Reflection loss of the symmetrical layer

5. RESULTS AND CONCLUSIONS.

We have considered a symmetrical layer of water with suspended solid particles: size 100 µm, density 2650 kg/m^3, compressibility 1.5x10^{-11} Pa^{-1}, in a volume concentration of 10%. The reflection coefficient calculate according to above equations with eight sub-layers of thickness d coating the central layer of thickness D is shown in the figure 2.

It can be concluded that the frequency response of the symmetrical layer show several arcs with minimum of reflection coefficient dues to the acoustic resonance :

a) when the thickness D is equal to a multiple of the half wavelength.
b) when the thickness d is equal to a multiple of the half wavelength.

The number of arcs repeated in the figure between the highest maximum is the number of the sub-layers

The absorption reduce the maximum-minimum amplitude. First those the central layer where the thickness is bigger and last those the lateral sub-layers.

The frequency of the first minimum due to the sub-layers increase when the gradient increase (D/d increase).

6. References

1. L.M. Brekhovskikh, *Waves in layered media* (Academic Press, New York, 1980).
2. Carbó R. Wave reflection from a transitional layer between the sea water and the bottom. *Jour. Acoust. Soc. Am.* **101** (1997), pp. 227-232.
3. Carbó R. Acoustical behaviur of the uppermost sea bottom layer. *Acustica united with Acta Acustica* **82**, (1996), pp.253.
4. A.J. Robins, *Reflection of plane acoustic waves from a layer of varying density,* Jour. Acust. Soc. Am. **87** (1990), p. 1546.
5. A.J. Robins, *Reflection of a plane wave from a fluid layer with continuously varying density and sound speed,* Jour. Acoust. Soc. Am. **89** (1991), p. 1686.
6. Fokin V.N. and Fokina M.S. A stepwise reconstruction of the characteristics of a layered elastic bottom from the reflection coefficient. *Acoustical Physics* **44** (1998), pp. 676-682.
7. Guillon L and Lurton X. Backscattering from buried sediment layers: The equivalent input backscattering strength model. *Jour. Acoust. Soc. Am.* **109** (2001), pp. 122-132
8. R.J. Urick, *The absorption of sound in suspensions of irregular particles,* Jour. Acoust. Soc. Am. **20** (1948), p. 283.
9. A.W. Wood, *A Textbook of Sound,* (Bell London, 1957).
10. M. Biot, *Theory of propagation of elastic waves in a fluid-saturated porous solid. I Low-frequency range, and II Higher-frequency range,* Jour. Acoust. Soc. Am. **26** (1956), p. 168.
11. L Landau and E. Lifshitz, *Mechanics of Continuous Media,* (Gostckhizdat, 1944).