INTERPOLATION OF LOW-ORDER HRTF FILTERS USING A ZERO DISPLACEMENT MEASURE

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ABSTRACT

Head-related transfer functions (HRTF) are generally measured at discrete azimuth and elevation values. Continuous motion of virtual sound images can only be obtained by interpolation of HRTFs for intermediate azimuth and elevation values in an immersive audio system. HRTFs can be modelled as a combination of low-order minimum-phase finite impulse response (FIR) filters and a constant delay line. A new method for interpolation of FIR filter zeros using a proximity-based vectorial displacement measure is proposed in this work.

INTRODUCTION

Head-related transfer function (HRTF) defines the spectral shaping of the sound signal on its way from a source location in the free field to the external ear. HRTFs are generally measured at discrete positions on a spherical grid equidistant from centre of the human head (or mannequin) whose measurements are being taken [1]. These measurements provide a set of HRTFs at discrete values of azimuth and elevation. The HRTFs provide a frequency dependent function of *interaural intensity difference* (IID) and a constant *interaural time delay* (ITD). The HRTF data obtained from these measurements are used for designing low-order digital filters to be made available for use in real-time audio spatialisation applications. A number of different approaches regarding the HRTF filter design process have been proposed [2][3][4].

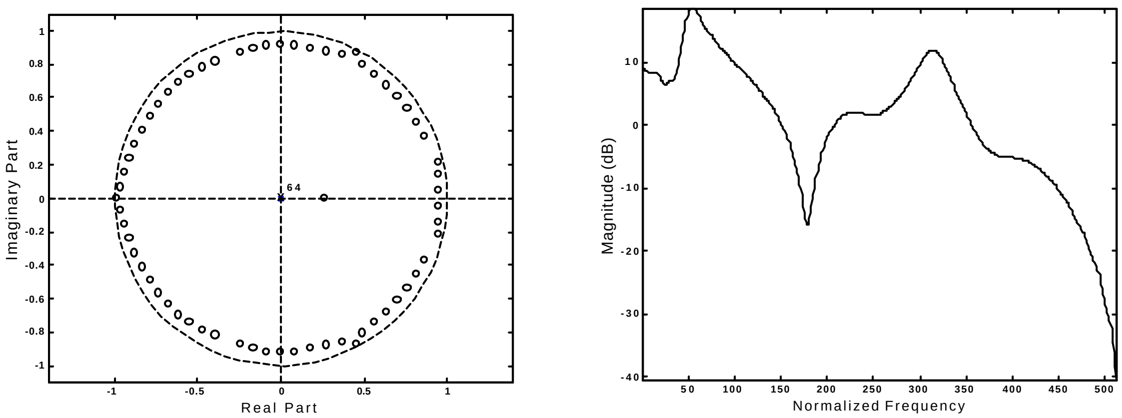
A virtual auditory display can be designed using the HRTF filters. For the virtual auditory display to be a *seamless* one, intermediate positions between the initially measured positions have to be synthesised as well. This synthesis can be considered, as an interpolation rather than extrapolation as HRTFs spanning a complete sampled space of different sound source locations are already available. This interpolation task requires a smooth transition of the spectral shape of the HRTF. This involves both the preservation of binaural cues and timbral properties of the auditory event. Smooth transition of filters enables a more realistic motion and more accurate positioning of the virtual sound images.

In this paper, a brief review of the HRTF filter design will be given first. Secondly, HRTF filter interpolation for FIR filters with a vectorial zero-displacement measure will be presented following a quick summary of other interpolation methods reported in the literature. A solution to a general problem caused by the real zeros of the system will be then given.

HRTF FILTER DESIGN

It is essential to smooth the magnitude spectra of the HRTF data before the filter design process in order to obtain low order filters. Different methods for spectral smoothing include smoothing with gamma-tone filterbanks [5], auditory weighting [6], rectangular windowing [7] and smoothing with wavelet transforms [2]. After the smoothing process, various generic methods can be used to design infinite impulse response (IIR) or finite impulse response (FIR) filters from a smoothed magnitude response.

The FIR filters used in this work were obtained from the KEMAR HRTF data [8][9] using the application of two methods. The magnitude responses of the HRTFs were smoothed using the redundant wavelet transform [2]. Then, a minimum-phase representation of the resulting function was obtained using real cepstrum analysis [10] in the time-domain. The first 64 samples of the resulting minimum-phase signal were directly used as the FIR filter coefficients resulting in 64^{th} order FIR filters. Although the filter order was slightly higher than previously reported research, this provided a uniform spacing of filter zeros (see Fig. 1) increasing interpolation accuracy. Therefore, it was more desirable to use 64^{th} order FIR filters for this purpose.



(a) (b)

Fig. 1 (a) The pole-zero plot of the HRTF FIR filter on the horizontal plane at 45° azimuth.
(b) The magnitude response of the same HRTF filter.

HRTF FILTER INTERPOLATION USING A ZERO-DISPLACEMENT MEASURE

The majority of the previously reported interpolation methods use the HRTFs on the functional level rather than dealing with filter roots [11][12][13]. These methods utilize linear [11][12], spline [11] or triangular [13] interpolation to interpolate a new magnitude spectrum between two or three magnitude spectra and design a new filter from the interpolated magnitude response. A more complex approach [14] uses a combination of HRTFs to derive general spatial frequency response surfaces to obtain interpolated HRTFs. These approaches can provide any combination of two HRTFs, but leave the burden of designing new filters from the interpolated HRTFs. This would cause difficulties for a real-time system, which would dynamically interpolate between two sound source positions. The approach in this work takes the HRTF interpolation problem as a filter-morphing problem, dealing directly with the zeros of the FIR filters.

The interpolation method proposed in this paper assumes the same number of complex and real zeros of both filters to be interpolated. Excessive real zeros are therefore coupled and substituted with equivalent complex poles. The following derivation is used for this task:

$$(z-a)(z-b)=(z-c)(z-c^*),\quad c=p+je,\quad a,b,p\in R^+,$$

where a and b are positive real zeros of the filter, c^* denotes the complex conjugate operation and ε is sufficiently small. When the equation above is evaluated at z=1 (i.e. ω=0), p is found as:

$$p = 1 - \sqrt{(1 - a - b + ab)}$$

This operation guarantees that the substituted zeros are inside the unit circle, not disturbing the minimum-phase property of the HRTF filter. The maximum error in the magnitude spectrum is less than 0.3dB when ε is selected as 0.001. The same operation can also be applied to a couple of zeros that are both negative, which will produce new negative *quasi-complex* zeros. The number of real zeros was decreased to 2 for all of the FIR filters. After this reduction, it is possible to represent the FIR filters as a combination of 62 complex zeros and 2 real zeros:

$$H_1 = H_{1c} \times H_{1R} = \left[\prod_{i=1}^{31} (z \pm z_{1c}(i))(z \pm z_{1c}(i)^*) \right] \times \left[\prod_{j=1}^{2} (z \pm z_{1r}(j)) \right]$$

The second part of the equation above refers to a second order filter, which has highpass, lowpass or bandpass characteristics depending on the signs of the real zeros. This second order filter imposes a different interpolation problem, so it is treated individually. The complex zeros of the FIR filters to be used in the interpolation are sorted and a zero displacement vector is calculated as:

$$\vec{m}(i) = z_{1c}(i) - z_{2c}(i), \; i = 1,2,...,62$$

where $\angle(z_{1c}(i)) > \angle(z_{1c}(i+1))$ and $\angle(z_{2c}(i)) > \angle(z_{2c}(i+1))$ (i.e. the complex zeros are sorted according to their angles with modulo π). Locations of individual complex zeros can be calculated using this zero displacement vector as:

$$z_{nc}(i) = z_{1c}(i) - k(i)m(i) = (1 - k(i))z_{1c}(i) + k(i)z_{2c}(i)$$

where k is either a constant between 0 and 1 (which results in linear interpolation of the poles) or an even function of i. The selection of k depends on the level of similarity of the interpolated filter to one of the filters used for that interpolation. Hence, the complex portion of the interpolated HRTF is formed as:

$$H_n(z) = \prod_{i=1}^{31} (z - (1 - k(i))z_{1c}(i) - k(i)z_{2c}(i))(z - ((1 - k(i))z_{1c}(i) - k(i)z_{2c}(i))^*) .$$

Figure 2 shows the interpolation process for the complex parts of the two different HRTFs obtained by varying a constant k linearly from 0 to 1 with intervals of 0.1.

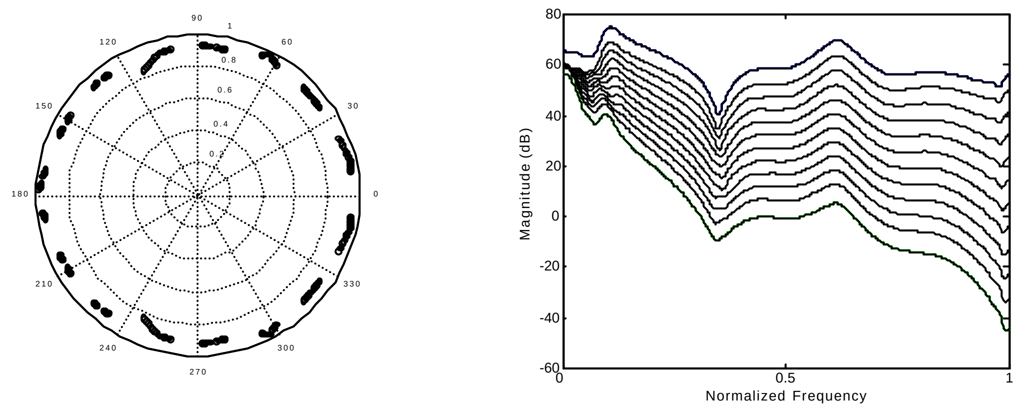


Fig. 2 (a) Movement of some of the zeros of the interpolated FIR filter for varying k, (b) Interpolated magnitude spectra of the complex poles of 45° azimuth (solid line on the top) and 55° azimuth (solid line on the bottom) in the horizontal plane (with 4dB separation)

It may be observed from the figure that, although the filter shape is strongly dependent on its complex zeros, the significant differences between any two consecutive filters are mainly due to their real zeros.

INTERPOLATION OF THE REAL ZEROS

The real zeros of the filters were interpolated separately. The negative real zeros of the FIR filter determine the filter shape at high frequencies, while the positive real zeros determine the shape at low frequencies. Preferably, the real zeros could also be interpolated using the method described above, but when the two filters contain different combinations of negative and positive real zeros, this becomes impossible.

The interpolation method for the real part of the filters deals only with the highest and lowest angular frequencies (i.e. ω=0 and ω=π). The derivation below is for the real parts of the two HRTF filters ($H_{1R}(z)$ and $H_{2R}(z)$), one of which contains two positive real zeros and the other contains two negative real zeros:

$$H_{1R}(z) = (z-a)(z-b),$$
$$H_{2R}(z) = (z+c)(z+d),$$

where $a,b,c,d \in R^+$. The aim of the interpolation is to obtain a new second order filter (i.e. $H_D(z)$) such that:

$$\log_{10}(H_D(z)) = l \times (\log_{10}(H_{1R}(z)) + \log_{10}(H_{2R}(z))),$$
$$0 < l < 1$$
$$H_D(z) = (z-m)(z+p)$$

where λ is a constant. The equations evaluated at *z=1* and *z=-1* yield:

$$((1-a)(1-b)(1+c)(1+d))^l - (1-m)(1+p) = 0,$$
$$((-1-a)(-1-b)(-1+c)(-1+d))^l - (-1-m)(-1+p) = 0$$

Positive results for the simultaneous solution to these equations yield *m* and *p*. Selection of the variable l determines the similarity of the interpolated second order filter to either one of the second order originator filters at higher and lower frequencies. Figure 3 shows the magnitude spectrum of the interpolated second order filter for l=0.5.

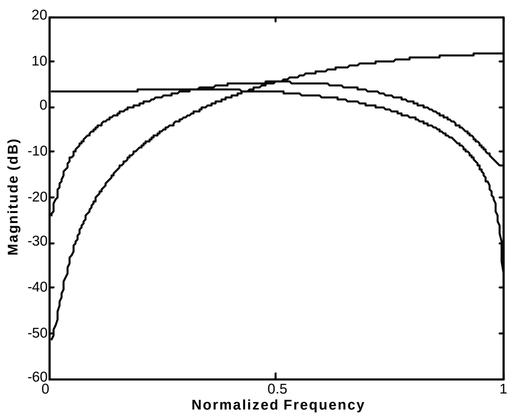


Fig. 3 Filters formed by the real zeros at 40° and 45° azimuth angles on the horizontal plane.

RESULTS AND DISCUSSIONS

It is possible to combine the two separate interpolation methods by coupling the k parameter of the first method and λ parameter of the second method and varying them linearly. Figure 4 shows the displacement of the complex zeros and the magnitude spectra obtained by such an interpolation between HRTFs of 45° and 55° azimuth in the horizontal plane.

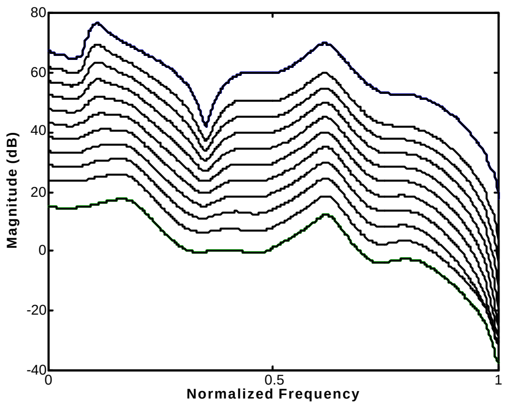


Fig. 4 Interpolation between HRTF filters for 45° azimuth (solid line on the top) and 55° azimuth (solid line on the bottom) in the horizontal plane for linearly increasing values of the coupled variables k and λ. (plotted with 5dB separation)

It may be observed from the figure that the interpolation method proposed in this paper is more like a morphing process from one HRTF filter to the other. The frequency notches and peaks, low frequency and high frequency details are captured and morphed correctly as well.

There is sufficient evidence in the literature that linear interpolation of HTRFs give sufficiently accurate results for azimuth angle intervals as large as 30° [12]. This makes smoothness of the timbral quality in the morphed filters a more important goal to achieve for a seamless positioning of the virtual sound images.

CONCLUSIONS AND FUTURE WORK

In this work, a new HRTF filter interpolation method for FIR filters based on the displacement of the filter's zeros was proposed. This method can be used for interpolation of the HRTF filters without the explicit information of the shape of the magnitude spectra of the originator HRTFs. The operations used in the derivation guarantee the minimum-phase property of the interpolated FIR filter. Since the interpolation of ITD is a rather straightforward task, it may be possible to use the method in a real-time immersive audio application.

The opportunities for future work include investigation of applicability of the current method to simple IIR filters, or common-pole IIR filters [3][15] which would provide a lower filter order and reduced computational complexity.

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