SOUND PROPAGATION IN A TURBULENT ATMOSPHERE: AN APPROACH USING SUBSTITUTE SOURCES

PACS: 43.28.Js, 43.28.Gq, 43.50.Gf

Forssén, Jens

Acoustics Research Centre
University of Salford
Salford M5 4WT
England
Tel: +46 161 295 3476
Fax: +46 161 295 5427
jf@ta.chalmers.se

ABSTRACT

The substitute-sources method (SSM) was previously implemented for a single noise barrier in a turbulent atmosphere by applying a substitute surface between the barrier and the receiver [1, 2]. Here, the method is extended, aiming to more general applicability to traffic noise propagation in urban environments. Examples are calculated for a turbulent atmosphere with upward refraction or without refraction. The results are compared with those from a parabolic equation method (PE) for the refractive cases and with an analytical solution otherwise. The results show good agreement, which indicates that the SSM could be useful for predictions of outdoor sound propagation.

1 INTRODUCTION

A substitute-sources method (SSM) was previously developed to predict the increased noise level behind a single barrier due to a turbulent atmosphere [1, 2]. The approach presented here aims to be more generally applicable. Of main interest is the prediction of noise propagation in urban environments, for instance for city planning purposes. In urban situations the propagation is expected to be influenced by many things: atmospheric turbulence, height varying sound speed profiles that may vary also with range, multiply reflecting and diffracting buildings and barriers, and varying ground properties. Parabolic equation methods (PE) are largely applicable to such situations [4, 5, 6]. Potentially applicable methods are those based on finite elements (FEM) or finite differences, boundary element methods (BEM) [7, 8] and fast field programs [9].

The approach with substitute sources presented here enables calculations for steeper geometries than the PE [2]. A steep geometry is for instance when a high barrier is located close to the source or to the receiver. The SSM models the propagation outward from the source, as also the PE does. This means that backscattering is neglected, unless calculated separately and added (as can be done in the PE [6]).

In the SSM, the sound field due to an original source is represented by a distribution of sources on a plane surface. The surface is called a substitute surface and the sources are called substitute sources, which can be seen as Huygens' secondary sources. Here, many substitute surfaces are put between the source and the receiver, with separation distances large compared with the wavelength. (See Figure 1.) The propagation is calculated in steps from one surface to the next for a non-turbulent atmosphere. The effect of turbulence is that it causes a loss in coherence of the sound field. Within each step the unperturbed, coherent field loses power into a residual, random field. The coherent field is further propagated toward the receiver and at each substitute surface the residual, random part is taken out. The contribution from different surfaces are uncorrelated, and the total power at the receiver is found by adding the power from the coherent field to the powers from the residual fields. A special mutual coherence function for the residual field is used.

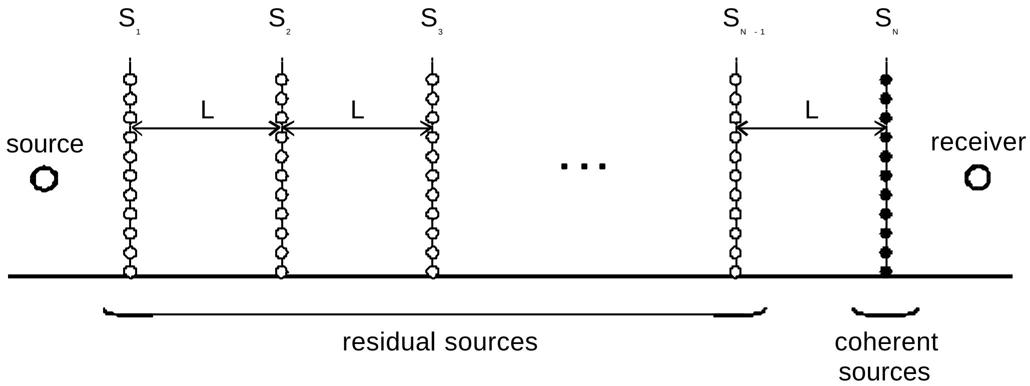


Figure 1. Substitute surfaces S_i , with separation distance L .

Since the calculation of the coherent field does not involve turbulence, many methods could be used. For instance ray-methods would be efficient for a homogeneous atmosphere or a linear sound speed profile. Here, a fast field program (FFP) is used throughout. The following section describes the theory and thereafter a few examples are calculated. The examples are for a hard and a finite impedance ground surface, with or without an upward refracting atmosphere. All calculations are for two-dimensional situations. A more detailed description of the theory and implementation is written in Ref. [3].

2 THEORY

2.1 Coherence in a Turbulent Atmosphere

The subject of line-of-sight propagation in a random medium has been studied extensively (e.g. [10]), and the theoretical results most useful here relate to the correlation between acoustic pressure signals that have travelled from monopole sources through different parts of the medium. In Figure (2) a geometry with two sources and two receivers are shown; $\boldsymbol{r}'$ and $\boldsymbol{r}$ are transversal separations and L is the longitudinal distance or range. For the case where the pressure p_1 is only due to source 1 and pressure p_2 is only due to source 2, the mutual coherence function (MCF) for p_1 and p_2 can be written as

$$\Gamma_{12} = \frac{\left\langle p_1 p_2^* \right\rangle + \left\langle p_1^* p_2 \right\rangle}{\left\langle \hat{p}_1 \hat{p}_2^* \right\rangle + \left\langle \hat{p}_1^* \hat{p}_2 \right\rangle} , \qquad (1)$$

where the complex conjugate is denoted by an asterisk (*), p_1 and p_2 are the fluctuating pressure amplitudes in the turbulent atmosphere and $\hat{p}_1$ and $\hat{p}_2$ are the amplitudes without turbulence (e.g. [11]). In the usual definition of the MCF there is only one source (i.e. coinciding source positions in Figure 2). This MCF is here referred to as $\Gamma^0(\boldsymbol{r}, L)$, where $\boldsymbol{r}$ is the distance between the receivers and L is the range. The reciprocal problem has the same MCF, i.e. when there are two sources and one receiver.

The extinction coefficient, g , is related to the decay over distance of the mean field in a turbulent atmosphere. If the pressure amplitude due to a point source in free field is $\hat{p}$ without turbulence, then the mean amplitude in turbulence will be $p_c = \left\langle p \right\rangle = \hat{p} \exp(-gL)$, where L is the distance of propagation [10]. The mean pressure amplitude, p_c , is also called the coherent field. The total field is the sum of the mean field and the residual, fluctuating field, $p = \left\langle p \right\rangle + p_r$, with $\left\langle p_r \right\rangle = 0$.

2.2 Coherence of the Residual Field

In Figure (3) a situation is described where only a part, L', of the range of propagation is through turbulence. This case could be formulated as

$$\left\langle |p_{\text{tot}}|^2 \right\rangle = |\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2|\cos[\arg(\hat{p}_2/\hat{p}_1)]\Gamma', \qquad (2)$$

where Γ' is the MCF for a turbulent layer with thickness L'. If $\boldsymbol{r}' << L' + L$, the propagation distance through the turbulence will be approximately L', and then the decrease in power in the coherent field can be approximated as by the factor $\exp(-2gL)$, using the above definition of the extinction coefficient. (Here, $\frac{1}{2}|p|^2$ is referred to as the power.) At the receiver, the contribution due to the coherent field can then be written as

$$\left\langle |p_c|^2 \right\rangle = \mathrm{e}^{-2gL'}\left(|\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2|\cos[\arg(\hat{p}_2/\hat{p}_1)]\right). \qquad (3)$$

Since the coherent pressure field, p_c, and the residual pressure field, p_r, are uncorrelated, one gets $\left\langle |p_{\text{tot}}|^2 \right\rangle = \left\langle |p_c|^2 \right\rangle + \left\langle |p_r|^2 \right\rangle$, and the residual contribution is given by:

$$\left\langle |p_r|^2 \right\rangle = \left\langle |p_{\text{tot}}|^2 \right\rangle - \left\langle |p_c|^2 \right\rangle = \left(1 - \mathrm{e}^{-2gL'}\right)\left(|\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2|\cos[\arg(\hat{p}_2/\hat{p}_1)]\frac{\Gamma' - \mathrm{e}^{-2gL'}}{1 - \mathrm{e}^{-2gL'}}\right). \qquad (4)$$

From the above equation a MCF for the residual field, $\tilde{\Gamma}$, can be generally defined as

$$\tilde{\Gamma} = \frac{\Gamma^0 - \mathrm{e}^{-2gL}}{1 - \mathrm{e}^{-2gL}}. \qquad (5)$$

The equations (3–5) can be seen as describing the transfer of the coherent field into a random field and how the contribution from the random field is calculated.

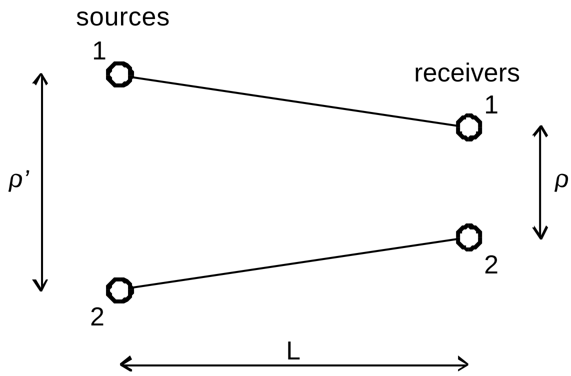


Figure 2. A pair of sound rays with transversal separation $\boldsymbol{r}'$ at the start and $\boldsymbol{r}$ at range L.

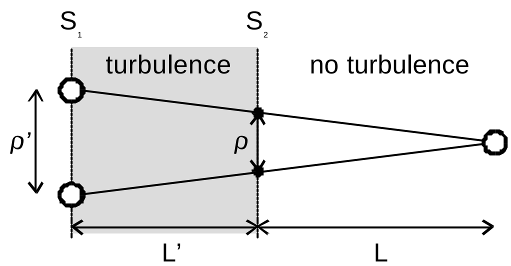


Figure 3. Propagation through a turbulent layer.

2.3 Numerical Model

In the SSM the sound field is represented by a distribution of sources on each substitute surface. For each step, from one substitute surface to the next, the power of the coherent field is reduced by the factor $\exp(-2gL)$, and the fraction of the power $[1 - \exp(-2gL)]$ is transferred to the residual field. The residual contributions from all but the last surface are calculated using the MCF $\tilde{\Gamma}$ for all source pairs. The last surface contributes with the remaining coherent field.

To describe the turbulence, a homogeneous and isotropic turbulence is assumed, that is, the fluctuations are assumed to follow the same statistics for all points and the statistics are independent of rotation. In the Gaussian turbulence model the medium fluctuations are described by a Gaussian spectrum. All the above assumptions simplify the turbulence modelling but this may be improved in future implementations of the SSM. For the Gaussian turbulence model the mutual coherence function for plane waves can be written

$$\Gamma^{\mathrm{pl}}(\boldsymbol{r}, L) = \exp\left(-2\gamma L\left[1-\exp\left(-\boldsymbol{r}^2/\ell^2\right)\right]\right), \qquad (6)$$

where ℓ is the correlation length of the fluctuations and $\gamma = \sqrt{\pi}k^2\ell\mu_0^2/2$, where μ_0^2 is the variance of the index of refraction fluctuations [12, 10]. In the calculations with the SSM presented here, only the plane wave MCF is used. The motivation is that between one substitute surface and the next, most pairs of rays toward the receiver can be approximated as parallel. The MCF for the residual field is then calculated as $\tilde{\Gamma} = [\Gamma^{\mathrm{pl}} - \mathrm{e}^{-2\gamma L}]/[1-\mathrm{e}^{-2\gamma L}]$.

3 RESULTS

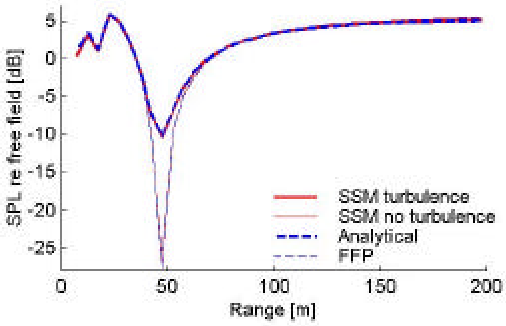
A few examples are calculated to study the behaviour of the SSM, involving upward refraction and no refraction for a hard or a soft, grass-like ground surface. The Green functions without turbulence are calculated using a fast feld program (FFP) implemented according to Salomons [13]. In the FFP the sound field is transformed into a wave number domain, which is used to efficiently calculate the velocities. In order to separate the direct and the ground reflected waves, the FFP calculations are made both with and without a ground surface. Subtracting the two results gives the ground reflected wave and the calculation without the ground gives the direct wave. Concerning computational demands, the SSM and the PE are fairly similar. In the SSM the computation time is dominated by the FFP calculations, i.e. by calculating the coherent velocity field.

For the cases with upward refraction, a logarithmic effective sound speed profile is used up to 30 m height. Above that height the sound speed is taken as constant to improve the numerical stability of the FFP. The logarithmic sound speed profile is $c(y) = c_0 - b\ln(y/y_{\mathrm{rough}}+1)$, with $b =$ 0.43 m/s and the roughness height $y_{\mathrm{rough}} =$ 0.05 m [13]. For the turbulence the correlation length $\ell =$ 1 m and the variance of the index of refraction $\mu_0^2 = 2 \cdot 10^{-6}$ or $5 \cdot 10^{-5}$ are used. The larger value for μ_0^2 models a strong turbulence and is chosen for the examples without refraction to give a strong turbulence effect at relatively short propagation range.

For upward refraction a shadow region is formed and, in general, a ray model will not work. In the SSM, however, the approach is different since the field due to the original source is substituted by a surface of sources. The sources at large enough height will be above the limiting ray of the shadow region and will thereby have direct rays to the receiver. Hence, it is assumed that the dominating contribution comes from the substitute sources that are above the limiting ray to the receiver.

The discretisation in height in the SSM, i.e. the distance between the discrete substitute sources is $\lambda/5$ for the non-refractive cases and $\lambda/10$ for the cases with upward refraction, where λ is the sound wavelength. These values were found from numerical tests without including the turbulence effects. The height used for the substitute surfaces is about half the maximum propagation range, and the top third is windowed to give a smooth decay with height of the source strengths. The windowing is used to reduce spurious oscillations in the solution [1]. The separation in range between the substitute surfaces, L, is 10 m in all examples, and the results are calculated every 5 meters. The calculations for the soft ground uses a normalised ground impedance of $3.71 + \mathrm{j}3.68$ at the frequency $f =$ 500 Hz. This value is for a grass-like surface and comes from the Delany and Bazely model using a flow resistivity of 200 kNs/m^4.

The calculated results are shown in Figures (4–7). Figures (4) and (5) are for a non-refractive atmosphere, for hard and soft ground. Figures (6) and (7) are for upward refraction, for hard and soft ground. All results shown here are for the frequency $f =$ 500 Hz. (Other data are given in the Figure captions.) The results are plotted as sound pressure level relative to free field as a function of propagation range from the source.



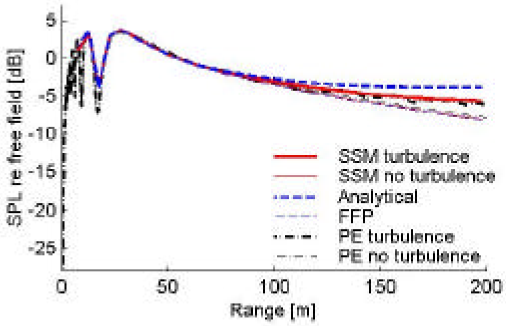
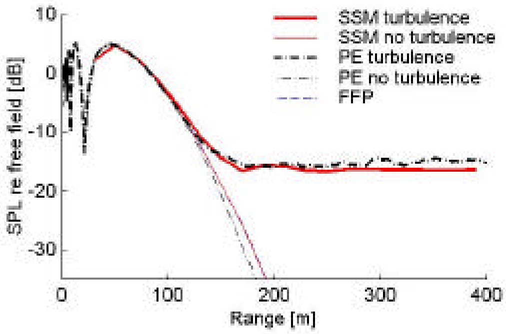


Figure 4. Result for hard ground, $m_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m (source height), $h_R = 4$ m (receiver height).

Figure 5. Result for soft ground, $m_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m, $h_R = 4$ m.



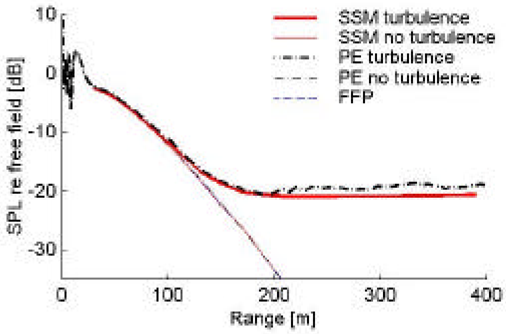


Figure 6. Result for hard ground and upward refraction, $m_0^2 = 2 \cdot 10^{-6}$, $h_S = 2$ m, $h_R = 2$ m.

Figure 7. Result for soft ground and upward refraction, $m_0^2 = 2 \cdot 10^{-6}$, $h_S = 2$ m, $h_R = 2$ m.

For the calculations without refraction (Figures 4 and 5) the two thicker curves show the results for a turbulent atmosphere. The solid line is for the SSM and the dashed line is the analytical solution using the MCF for the direct and ground reflected rays from the source to the receiver. The two thinner curves are for no turbulence; the dashed line for the FFP directly and the solid line for the SSM without turbulence, i.e. where all power is calculated from the last substitute surface without decorrelation. (These two curves are almost indistinguishable in the Figures.) In the examples with upward refraction (Figures 6 and 7), a comparison is made with a parabolic equation method (PE). In the PE calculations 50 realisations of the turbulence are used to estimate the power at the receiver. The dash-dotted curves show the PE results, with and without turbulence.

For an atmosphere without refraction, the effect of turbulence is mainly that is reduces the interference, as can be seen in Figures (4) and (5). The SSM result for hard ground shows good agreement with the analytical, whereas for the soft ground (Figure 5) there is a significant discrepancy. To provide an additional comparison, the PE method is applied also to this case, and the results are shown in the same figure. The PE results are similar to those from the SSM, which indicates that the analytical solution may give a significant error in this case. A possible explanation is that the turbulence scattering results in a larger loss into the ground than what is given by the ray model. For upward refraction the main effect of the turbulence is that is limits the acoustic shadow, as can be seen in Figures (6) and (7). The results from the SSM are shown to agree well with the PE results.

4 CONCLUSIONS

The good agreement shown in the comparison with the other methods indicate that the extended substitute-sources method presented here could be a useful tool for predictions of outdoor sound propagation. The approach also enables application to steep geometries, as shown in a previous implementation for a single barrier [2]. In future work, range dependent properties of the atmosphere and the ground could be taken into account. For example, with small changes in the method, a ground impedance that is step-wise constant over range could be modelled.

ACKNOWLEDGEMENTS

The work presented here was made during a post-doctorial fellowship at the Acoustics Research Centre, University of Salford, UK. The author wishes to thank Professor Y. W. Lam for the invitation and hosting. The author also wishes to thank Wolfgang Kropp and Mikael Ögren for their help with preparing the manuscript. The fellowship was funded by STINT (The Swedish Foundation for International Cooperation in Research and Higher Education).

BIBLIOGRAPHICAL REFERENCES

[1] Forssén J. Calculation of noise barrier performance in a turbulent atmosphere by using substitute sources above the barrier. Acustica–Acta Acustica, Vol. 86, 2000, pp. 269-75.

[2] Forssén J. Calculation of noise barrier performance in a three-dimensional turbulent atmosphere using the substitute-sources method. Acustica–Acta Acustica, Vol. 88, 2002, pp. 181-189.

[3] Forssén J. An extended substitute-sources method for a turbulent atmosphere: Calculations for upward refraction. Submitted to Acustica–Acta Acustica, May 2002.

[4] Gilbert, K. E., Raspet, R. and Di, X. Calculation of turbulence effects in an upward refracting atmosphere. J. Acoust. Soc. Am., Vol. 87, 1990, pp. 2428-2437.

[5] Sack, R. A. and West, M. A parabolic equation for sound propagation in two dimensions over any smooth terrain profile: The generalised terrain parabolic equation (GT-PE). Applied Acoustics Vol. 45, 1995, pp. 113-129.

[6] West, M., and Lam, Y. A Two Way Vertical Interface Parabolic Equation (TV-PE) Model for Atmospheric Propagation in the Presence of Severe Terrain Features. Proc. 9th Int. Symp. on Long-Range Sound Propagation, The Hague, Netherlands, 2000, pp. 167-177.

[7] Premat, E. and Gabillet, Y. A new boundary-element method for predicting outdoor sound propagation and application to the case of a sound barrier in the presence of downward refraction. J. Acoust. Soc. Am., Vol. 108, 2000, pp. 2775-2783. 7

[8] Salomons, E. M., Geerlings, A. C., and Duhamel, D. Comparison of a ray model and a fourier-boundary element method for traffic noise situations with multiple diffractions and reflections. Acustica–Acta Acustica, Vol. 83, 1997, pp. 35-47.

[9] Taherzadeh, S., Li, K. M., and Attenborough, K. A hybrid BIE/FFP scheme for predicting barrier efficiency outdoors. J. Acoust. Soc. Am., Vol. 110, 2001, pp. 918-924.

[10] Ostashev V. E. Acoustics in moving inhomogeneous media. E & FN Spon (an imprint of Thomson Professional), London, 1997.

[11] Salomons, E. M. The fluctuating field of a monopole source in a turbulent atmosphere above a ground surface. Time-averaged sound pressure level and statistical distributions. Proc. 8th Int. Symp. on Long-Range Sound Propagation, The Pennsylvania State University, 1998, pp. 326-351.

[12] Ostashev, V. E., Mellert, V.,Wandelt, R. and Gerdes, F. Propagation of sound in a turbulent medium. I. Plane waves. J. Acoust. Soc. Am., Vol. 102, 1997, pp. 2561-2570.

[13] Salomons, E. M. Computational atmospheric acoustics. Kluwer Academic Publishers, Dordrecht, 2001.