A FINITE ELEMENT MODEL FOR SOUND PROPAGATION IN COMPLEX CITIES

Pontus J. Thorsson
Department of Applied Acoustics, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden
Tel: +46 31 772 2214, Fax: +46 31 772 2212
E-mail: pontus@ta.chalmers.se

PACS: 43.28.Js

Abstract

Sound propagation in complex acoustic environments such as cities is in this paper studied by using the Boltzmann equation for stochastic molecular motion in a perfect gas. The equation is solved using a finite element approach, where the city is divided into its relevant parts such as roads, different building structures and recreational areas. Each part is characterized through its mean building size, mean absorption properties and the density and frequency spectrum of the sources present. The results from this method are not pointwise exact, but instead intended as a mean value of the background noise level.

1 INTRODUCTION

Noise propagation from simple sources is a well-studied subject in acoustics. A multitude of analytical models exist for calculating the noise levels from monopoles (see for example [1]). The models include different grades of complexity, e.g., diffraction around corners, finite ground impedance or a varying ground profile. However, the major part of these models only include a single source. In principle it is possible to superimpose the fields from several sources, but this can be a tedious assignment, since the geometry must be given in relation to each source.

Previous studies have shown that common prediction methods can give substantial errors in shielded situations [2]. This is probably caused by the fact that only the nearest road is normally assumed to contribute to the noise level. Especially in strongly shielded situations this hypothesis would be incorrect.

In a real city there are often a vast amount of sources, e.g., different kinds of fans and sources related to vehicle transportation. It is not feasible to handle this large number of individual sources separately with analytical methods. Numerical methods, like the Finite Element Method (FEM), are better adapted to this situation. However, when considering outdoor noise propagation the sources that are judged to be of importance are often distributed over a large area, which causes the numerical model to get very large in terms of storage and solution time. The calculation time is also rapidly increasing with frequency, since the element size in the discretization is linked to the wavelength. A possible alternative is then to treat the noise propagation as a form of diffusion or transportation process. By using these kinds of models it is possible to use elements of larger size, and hence it is possible to calculate over larger areas. A simple diffusion model can be achieved through applying the heat equation

$$\frac{\partial u}{\partial t} - k\Delta u = 0 \ , \tag{1}$$

to the propagation problem. This approach gives however not accurate results [3]. Better results can be achieved with a method based on transport, such as the Boltzmann collison equation for molecular dynamics in a perfect gas. Here this equation has been applied to sound propagation in cities. In transport applications sound is assumed to be transported in small packages; sound particles or phonons.

2 THEORETICAL BASIS OF THE METHOD

The Boltzmann collision equation is normally used for calculation of molecular dynamics. In order to use it in acoustics some modifications are needed. In its most general form the Boltzmann equation can be written as:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{1}{m}\mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{v}} = J_{gain} - J_{loss}. \tag{2}$$

This equation describes the dynamics of the particles inside the element $\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}$ in phase space, where $\mathbf{r} = (x, y, z), \mathbf{v} = (v_x, v_y, v_z)$). The distribution function f describes the density of particles in phase space. The vector $\mathbf{F}$ describes any forces acting on the particles, and in most acoustical problems it can be set to zero. The value on the right hand side corresponds to the net flow of particle densities in and out of the element $\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}$.

The main difference between the Boltzmann equation for acoustics and for molecular dynamics lies in the right hand side. In molecular dynamics the molecules collide and are hence scattered, while in acoustics sound particles superimpose onto eachother, which gives no inter-particle scattering.

The Boltzmann equation for noise transport with isotropic scattering can be derived as (from [3]):

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{c}{\lambda} f = \frac{1-\alpha}{2\pi\lambda} \int f \ \mathrm{d}\mathbf{v}. \tag{3}$$

In this equation $|\mathbf{v}| = c$ is the speed of sound and λ is the mean free path length of the sound particles. In the derivation of this equation, the propagation distance for the phonons without colliding with an obstacle is assumed to obey a negative exponential distribution. The mean free path length for the phonons can be calculated as

$$\lambda = \frac{1}{nQ}, \tag{4}$$

where n is the density of obstacles, i.e., houses or trees, and Q is the average scattering cross section for the obstacles.

The phonons are assumed to propagate over a flat plane loosely built with houses. The houses are assumed to be random in position, orientation and size, but the probability distributions for these parameters should still give a meaningful average inside a given domain. A city can be thought of as a set of domains with different parameters. Each domain represents an area of specialized building density or building size and can be used to characterize different kinds of urban or suburban city environments, including parks or recreation areas. However, the house concentration

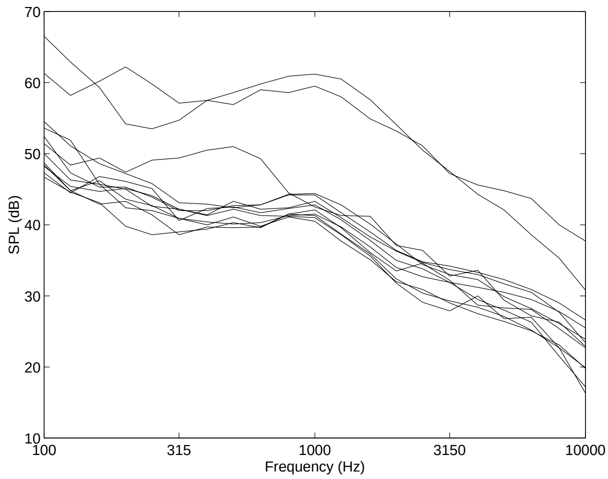

Figure 1: Sound pressure level as a function of frequency at Hägerstensvägen, Stockholm. The two upper curves correspond to directly exposed positions, while the other curves represent positions with varying degrees of shielding.

is restricted to be relatively low. This restriction comes from the definition of the infinitesimal element in phase space ($\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}$) that has been used in the derivation of the Boltzmann equation (see [4] for details).

The concept of mean free path length is relying on constant statistical parameters for the whole domain, i.e., the city. Thus Eq. 3 is not exact for varying parameters, but since the distance involved in sound propagation through cities often is large compared to the value of the mean free path length it is possible to use this approach as a first approximation. Certainly the results close to the boundaries of the subdomains will be inaccurate, but since the aim of the present model is to achieve an average background noise level this drawback is not judged to be of critical importance. The validity of this approach will be studied in further work with the present model.

Eq. 3 can be further simplified by assuming the geometry to be two-dimensional, by assuming steady-state condition, and by replacing the velocities in x- and y-direction with polar representations. All sound particles travel with the speed of sound c and only the direction of propagation ϕ is hence of importance. This leads to the simplified equation:

$$\frac{\partial f}{\partial x}\cos\varphi + \frac{\partial f}{\partial y}\sin\varphi + \frac{f}{\lambda(\mathbf{r})} = \frac{1-\alpha(\mathbf{r})}{2\pi\lambda(\mathbf{r})}\int_0^{2\pi} f \, \mathrm{d}\varphi. \qquad (5)$$

In this equation $f = f(\mathbf{r}, \varphi)$, where $\mathbf{r} = (x, y)$. Interesting is that this equation is not dependent on the speed of sound.

The present model can be used to calculate the average noise levels in a part of a city. The boundary conditions for the region of interest should be adapted to that situation. Neumann boundary conditions are used in the x- and y-direction, since the calculation domain boundaries have no fixed noise levels. Because of the definition of φ as an angular direction, cyclic boundary conditions ($f(\mathbf{r}, 2\pi) = f(\mathbf{r}, 0)$) are used in the φ-direction.

There is no direct dependency on frequency in Eq. 5, but the frequency can affect the results indirectly through the parameters λ and α. However, noise levels measured close to roads and in shielded areas have shown relatively constant frequency contents. Figure 1 shows measured results for various degrees of shielding, from directly exposed positions to positions inside closed courtyards, at Hägerstensvägen in Stockholm. The frequency spectrum indicates that it may be possible to calculate noise levels in cities without regarding frequency dependency. The possibility of this assumption will be studied in further work.

3 SOLUTION METHOD

In this section a solution method for Eq. 5 based on the variational finite element method will be outlined. The used method is often referred to as continuous Galerkin method of order 1 (cG1) [5]. Both sides of Eq. 5 are multiplied with a known continuous test function $\Phi(\mathbf{r},\varphi)$ and integrated over the whole domain of interest Ω. The domain Ω is now discretized in a three-dimensional mesh. For simplicity, the 3D mesh is derived from a 2D mesh by translation in the third dimension. By using a fixed discretization in the third dimension, the volume is built up from prism-shaped elements like in Figure 2. It is also possible to achieve tetrahedral elements by using a translation procedure, but this is more cumbersome. An example of a 2D mesh for a part of the city of Stockholm is shown in Figure 3. The area is roughly 2.2×3.2 km and the final 3D mesh holds 5901 nodes and 9702 prism elements. The test functions used here are first order polynomials which are adapted to the used elements. For one single node (x_j, y_j, φ_j) the test function can be written as:

$$\Phi_j(x,y,\varphi) = \left(1 - \frac{|x-x_j|}{h_x} - \frac{|y-y_j|}{h_y}\right)\left(1 - \frac{|\varphi-\varphi_j|}{h_\varphi}\right) \quad . \tag{6}$$

The parameters h_x, h_y and h_φ are the dimensions of the reference element in Figure 2. Each element is built up from six test functions:

$$\Phi(\mathbf{r},\varphi) = \sum_{j=1}^{6} \Phi_j(\mathbf{r},\varphi) \quad . \tag{7}$$

Using the test functions in Eq. 6 it is possible to write a discretized version of Eq. 5:

$$\sum_{i=1}^{N}\left[\int_{\Omega_i} \frac{\partial f}{\partial x}\cos\varphi\ \Phi\ \mathrm{d}\Omega_i + \int_{\Omega_i} \frac{\partial f}{\partial y}\sin\varphi\ \Phi\ \mathrm{d}\Omega_i + \int_{\Omega_i} \frac{f}{\lambda}\ \Phi\ \mathrm{d}\Omega_i\right]$$
$$= \sum_{i=1}^{N}\int_{\Omega_i} \frac{1-\alpha}{2\pi\lambda}\int_{\varphi=0}^{2\pi} f\ \mathrm{d}\varphi\ \Phi\ \mathrm{d}\Omega_i \quad . \tag{8}$$

The distribution function can now be replaced by its interpolant. The interpolant is built from basis functions like in Eq. 6:

$$f(\mathbf{r},\varphi) = \sum_{j=1}^{6} f_j\ \Phi_j(\mathbf{r},\varphi) \quad , \tag{9}$$

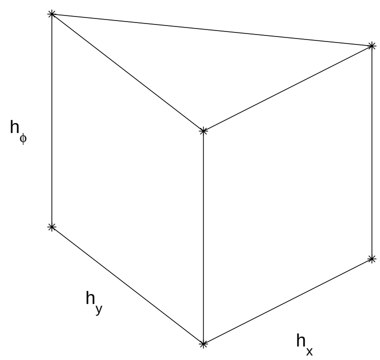

Figure 2: A prism-shaped reference element built up through a translation of a 2D mesh.

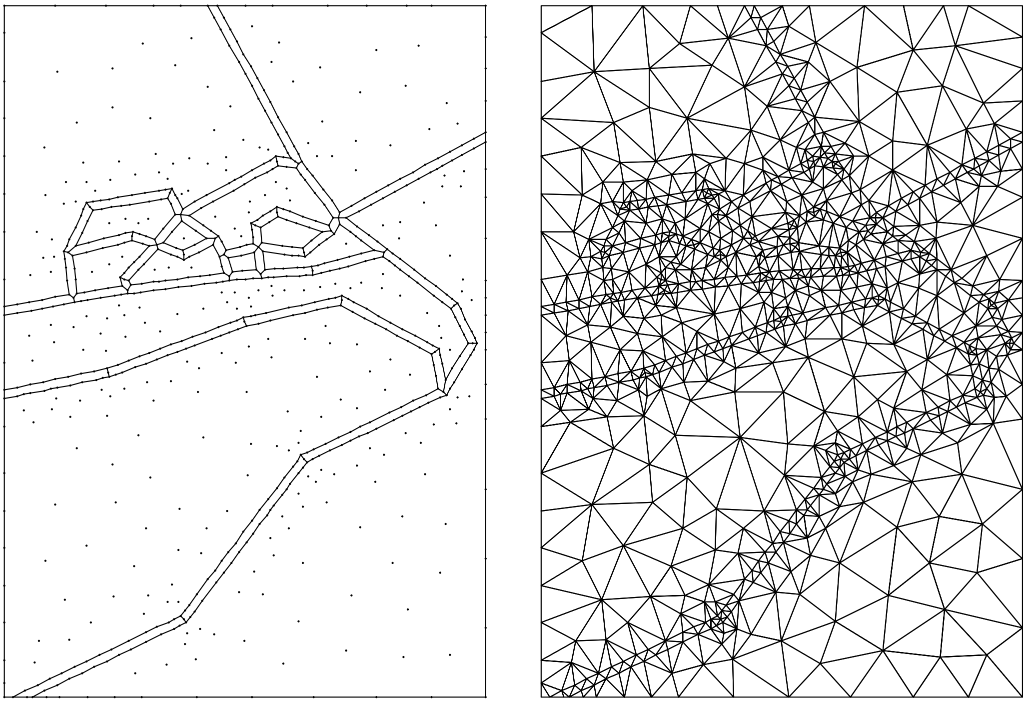

Figure 3: Left: A simplified figure of the roads important for the traffic noise around Hägerstensvägen in Stockholm. Right: A 2D mesh of the area (843 nodes and 1617 triangular elements).

where f_j is the value of the solution at node j. By inserting the expressions for the test functions (Eq. 6) and the interpolant for f (Eq. 9) an expression suitable for numerical evaluation is achieved:

$$\sum_{i=1}^{N}\sum_{j=1}^{6}\sum_{k=1}^{6} f_j \left[A_{i,j,k} + B_{i,j,k} + E_{i,j,k} - F_{i,j,k}\right] = 0 \ . \tag{10}$$

Here,

$$\begin{aligned}
A_{i,j,k} &= \int_{\Omega_i} \frac{\partial \Phi_j}{\partial x}\ \Phi_k \cos\varphi \ \mathrm{d}\Omega_i \ , \\
B_{i,j,k} &= \int_{\Omega_i} \frac{\partial \Phi_j}{\partial y}\ \Phi_k \sin\varphi \ \mathrm{d}\Omega_i \ , \\
E_{i,j,k} &= \frac{1}{\lambda}\int_{\Omega_i} \Phi_j \ \Phi_k \ \mathrm{d}\Omega_i \ , \\
F_{i,j,k} &= \frac{1-\alpha}{2\pi\lambda}\int_{\Omega_i} \Phi_k \int_{\varphi=0}^{2\pi} \Phi_j \mathrm{d}\varphi \ \mathrm{d}\Omega_i \ .
\end{aligned}$$

The integrals in Eq. 11 can now be evaluated analytically. The evaluations must be done by integrating for all different combinations of i,j and k. The contribution from each prism element in Eq. 8 can thus be interpreted as a 6×6 matrix of contributions to nodes. An efficient way do this numerically is to evaluate the integrals on a reference element and then map the reference element onto each element. The final equation system is assembled element by element using the submatrices described above. The equation system can easily be solved with standard methods.

The development of the present model is not finished, but initial tests have shown promising results and the calculation times are fairly short.

4 CONCLUSIONS AND FURTHER WORK

The present model is promising for calculating the average background noise level in a part of a city. The approach by using a transport equation instead of the wave equation presents possiblities to calculate over larger areas, like cities. The basic mathematics of a FEM solution of the Boltzmann collision equation have been presented in this paper. The model is under development and no results are therefore available at the moment. In future work the model will be used to study the importance of long-distance sound propagation from distributed sources like roads. The results from this model will also be used in combination with short-range models to predict noise levels in cities.

5 ACKNOWLEDGMENTS

This paper is based on a study performed within the research programme ”soundscape support to health”, sponsored by the Swedish Foundation for Strategic Environmental Research (MISTRA), the Swedish Agency for Innovation Systems (Vinnova) and the Swedish National Road Administration (VV).

References

[1] Jonasson H. and Nielsen H. L. (1996). Road Traffic Noise – Nordic Prediction Method. ISBN 92 9120 836 1. TemaNord 1996:525, Nordic Council of Ministers.

[2] Ögren, M. and Forssén, J. (2001). Prediction of noise levels in shielded urban areas, Proceedings of Inter-Noise 2001, The Hague, Netherlands.

[3] Kuttruff, H. (1982). A mathematical model for noise propagation between buildings. Journal of Sound and Vibration **85(1)**, pp. 115–128.

[4] Sommerfeld, A. (1971). Thermodynamics and statistical mechanics. Academic Press, London.

[5] Eriksson, K., Estep, D., Hansbo, P. and Johnson, C. (1996). Computational differential equations. Studentlitteratur, Lund, Sweden.