CHARACTERISATION OF A SERVO STEERING PUMP AS A SOURCE OF STRUCTURE BORNE SOUND

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ABSTRACT
The aim of this study is the prediction of the power transmission from the dynamic characteristic of the uncoupled subsystems to furnish the sound power fed to the recipient. In order to qualify the concept of interface mobility with respect to automotive accessory applications involving multi-point, multi-component sources an experiment is undertaken with a servo-steering pump as a source of structure borne sound. The complex interface mobilities of both source and receiver were determined. The velocities of the operating pump were measured at the three mount points when dynamically free. As a reference, the transmitted power was additionally calculated from the kinetic energy of the receiver, directly driven by the operating pump.

1. INTRODUCTION

The sound and vibration transmission between structural elements are frequently the focus of noise and vibration control. In many engineering situations sources of sound and vibration structural elements are mounted on flexible structures. The sources, for example, generators, pumps, etc. when in operation causes unwanted sound and vibration close to or remote from the excitation.
The vibration transmission from the source to the receiving structure is governed by the source strength, together with the dynamic characteristic of the source, the receiver and any coupling elements.
The aim of this study is the prediction of the power transmission from the dynamic characteristic of the uncoupled subsystems to furnish the sound power fed to the recipient. Therefore, it is often necessary to predefine target levels for both the source and the receiver such that the dynamics of these substructures are mismatched to each other [1].
Often, the analysis relies on the assumption of point-like connection for which the methodology is comparatively well developed [2]. Computations and measurements increase, however, already for a limited number of connection points so simplifications are generally necessary.
Sometimes the force exerted by the source is used to define the transmission, based on the assumption of rigid body motion on ideal spring mounts on a very low mobile foundation [3]. In practice, this assumption is only valid for low frequencies.
A general representation of the power transmission by the source strength and the dynamic characteristics of source and receiver is proposed in reference [4], in case of one contact point and one component of motion. A similar approach for the case of source and receiver beams connected via an isolator element can by found in [5].

In reference [6] the vibrational power is regarded as the power transmitted by N independent poles of vibration. A series of experiments were conducted on a machine-isolator-seating arrangement up to 1000 Hz.
In references [7] and [8] the concept of interface mobility is introduced. With this concept a formal simplification can be obtained by transcribing the multi-point/ multi-component transmission problem into that of single point and single component.
It is found, that in the range in which the wavelength of the recipient is much longer then the typical cross section of the interface the interface mobility can be replaced by the ordinary point mobility. For shorter wavelengths the interface mobility is also dependent on the interface geometry. Anyhow, in view of the orthonormal basis for the interface mobility, the simplicity of the single-point, single-component case can be retained and terms included as required.

Figure 1: Servo steering pump equip with three rubber isolators mounted upon three steel stands.

In order to qualify this technique with respect to automotive accessory applications involving multi-point, multi-component sources, an experiment is undertaken with a servo-steering pump as the source of structure borne sound. The transmission problem is exemplified with the pump mounted on specially manufactured supports, bolted to a plywood plate, the latter representing an arbitrary recipient structure. The complex interface mobilities of both source (the servo steering pump) and receiver (the plywood plate) were determined in terms of the uniform and the first order interface mobility. The velocities of the operating pump were measured at the three mount points when dynamically free. As a reference, the transmitted power was additionally calculated from the kinetic energy of the receiver, directly driven by the operating pump, and alternatively from the axial component of force and velocity obtained at the three mount points.

2. INTERFACE WITH THREE POINT-LIKE CONTACTS

The servo steering pump was mounted on the receiver via three point-like contacts. In this case the velocities and the forces are defined by the two vectors

$$v_{sf}(s)=\begin{bmatrix} v_{a,1} \\ v_{a,2} \\ v_{a,3} \\ v_{r,1} \\ v_{r,2} \\ v_{r,3} \end{bmatrix},\; F(s_0)=\begin{bmatrix} F_{a,1} \\ F_{a,2} \\ F_{a,3} \\ F_{r,1} \\ F_{r,2} \\ F_{r,3} \end{bmatrix},\; Y^{S,R}(s|s_0)=\begin{bmatrix} Y^{a,1}_{a,1} & Y^{a,1}_{a,2} & Y^{a,1}_{a,3} & Y^{a,1}_{r,1} & Y^{a,1}_{r,1} & Y^{a,1}_{r,1} \\ Y^{a,2}_{a,1} & Y^{a,2}_{a,2} & Y^{a,2}_{a,3} & Y^{a,2}_{r,1} & Y^{a,2}_{r,2} & Y^{a,2}_{r,3} \\ Y^{a,3}_{a,1} & Y^{a,3}_{a,2} & Y^{a,3}_{a,3} & Y^{a,3}_{r,1} & Y^{a,3}_{r,2} & Y^{a,3}_{r,3} \\ Y^{r,1}_{a,1} & Y^{r,1}_{a,1} & Y^{r,1}_{a,1} & Y^{r,1}_{r,1} & Y^{r,1}_{r,1} & Y^{r,1}_{r,1} \\ Y^{r,2}_{a,1} & Y^{r,2}_{a,2} & Y^{r,2}_{a,3} & Y^{r,2}_{r,1} & Y^{r,2}_{r,2} & Y^{r,2}_{r,3} \\ Y^{r,3}_{a,1} & Y^{r,3}_{a,2} & Y^{r,3}_{a,3} & Y^{r,3}_{r,1} & Y^{r,3}_{r,2} & Y^{r,3}_{r,3} \end{bmatrix} \quad (2.1)$$

and the mobilities $Y^{S,R}$ of the source (S) and the receiver (R) are defined by the associated (6x6)-matrices, respectively.

After the first preliminary investigation the measurement effort was increased and the axial and the radial component were included, so the velocity and force were represented by vectors with six components, three axial v_a , F_a and three radial v_r , F_r .

This expands the mobility matrixes to 6x6 with four 3x3 sub matrices. The first sub matrix, of course, represents the coupling of the axial component, with the direct path in the main diagonal. The fourth represents corresponding the radial component of motion. The second and third sub matrices refer to the coupling between different components where, again, the main diagonal represents the coupling at the point under study.

The real part of the complex power transmitted can be obtained as [9],

$$W = \frac{1}{2}\mathrm{Re}\left\{F^{*T} v^{R}\right\} \qquad (2.2)$$

where, in the case of three contact points, the force and velocity are the aforementioned vectors. Upon introducing the mobility matrix of the receiving structure this expression becomes [4],

$$W = \frac{1}{2}\mathrm{Re}\left\{F^{*T} Y^{R} F\right\} \qquad (2.3)$$

With the expression for the forces at the interface substituted, the complex power transmission is obtained in terms of the free velocity and the mobilities at the contact points as

$$W = \frac{1}{2}\mathrm{Re}\left\{\left\{\left[Y^{R*} + Y^{S*}\right]^{-1} v_{sf}^{*}\right\}^{T}\left\{Y^{R}\left(\left[Y^{R} + Y^{S}\right]^{-1} v_{sf}\right)\right\}\right\} \qquad (2.4)$$

The effort and requirements on measurements increase rapidly with the number of contact points. Moreover, no information is revealed as to which transmission path is the most important.

3 INTERFACE MOBILITY

The interface mobility is defined as the Fourier series expanded response field at the interface to the associated, unit amplitude, component of the stress field [8], [10].

$$Y_q = \frac{v_q}{F_q}\,, \qquad (3.1)$$

where

$$v_q = \frac{1}{C}\int_0^C v(s)e^{-\frac{i2qps}{C}}ds\,,\; F_q = \frac{1}{C}\int_0^C F(s_0)e^{-\frac{i2qps_0}{C}}ds_0 \qquad (3.2)$$

and $$Y_{pq} = \frac{1}{C^2}\oint_C\oint_C Y(s\,|\,s_0)e^{-jk_p s}e^{-jk_q s_0}ds ds_0 \text{ with } k_p = \frac{p2p}{C},\; k_q = \frac{q2p}{C} \qquad (3.3)$$

where C is the perimeter of the continuous interface contour, which contains the three contact points and $v(s)$ and $F(s)$ the obtained velocity and force (theoretically smeared along the contour $F(s) = 1/C \sum F_m d(s - s_0)$).

For an *N*-point, discrete interface the velocity and force become the aforementioned vectors (Eq. 2.1) and the Fourier transformation of the continuous velocity and force functions *v(s), F(s)* become a Fourier series,

$$V(k) = \sum_{n-1}^{N} v(n)e^{-\frac{i2p(k-1)(n-1)}{N}},\, 1 \le k \le N\,,\; F(k) = \sum_{n-1}^{N} F(n)e^{-\frac{i2p(k-1)(n-1)}{N}},\, 1 \le k \le N \qquad (3.4)$$

In the above expression *v(n)* is the physical velocity vector, where the elements represent the velocity at each contact point. *V(k)* is the column vector, representing the zero and first orders of the motion. The vectors *F(k)* and *F(n)* are similarly defined, whereby the matrix $Y(s, s_0)$ requires a two-dimensional Fourier decomposition.

The velocity at any point along is given by,

$$v(s) = \oint_C Y(s \mid s_0) F(s_0) ds_0 \qquad (3.5)$$

Substituting the Fourier expansion of $Y(s \mid s_0) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} e^{jk_p s} e^{jk_q s_0}$ into Eq. (3.5) yields

$$v(s) = \oint_C \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} e^{jk_p s} e^{jk_q s_0} F(s_0) ds_0 = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} e^{jk_p s} \oint_C F(s_0) e^{jk_q s_0} ds_0 = C \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} e^{jk_p s} \tilde{F}_q \qquad (3.6)$$

where $\tilde{F}_q$ is the coefficient of the Fourier expansion of $F(s)$

$$\tilde{F}(s) = \sum_{q=-\infty}^{\infty} F_q e^{jk_q s} \; . \qquad (3.7)$$

By expanding $v(s)$ into Fourier series

$$\sum_{p=-\infty}^{\infty} v_p e^{jk_p s} = v(s) = C \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} e^{jk_p s} F_q \qquad (3.8)$$

and comparing the coefficients of the terms on both sides yields

$$v_p = C \sum_{q=-\infty}^{\infty} \hat{Y}_{qp} F_q \; , \qquad (3.9)$$

or in terms of vectors and matrix

$$\vec{v}_p = C \overline{\overline{Y_{pq}}} \vec{F}_q \qquad (3.10)$$

Substituting the expression of $v(s)$ and $F(s)$ the active power can be obtained as

$$W = \frac{1}{2} \int_C \mathrm{Re} \left\{ \sum_{q=-\infty}^{\infty} F_q^* e^{-jk_q s} C \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} Y_{qp} e^{-jk_p s} F_p \right\} ds = \frac{1}{2} C^2 \, \mathrm{Re} \left\{ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} F_q^* Y_{qp} e^{-jk_p s} F_p \right\} (3.11)$$

or in terms matrix and vectors

$$W = \frac{1}{2} C^2 \, \mathrm{Re} \left\{ \vec{F}(k_q)^* \overline{Y}(k_q, k_p) \vec{F}(k_p) \right\} \qquad (3.12)$$

The expression for the Fourier series of the force substituted the power transmitted through the interface is calculated by

$$W = \frac{1}{2} \mathrm{Re} \left\{ \left\{ \left[\overline{\overline{Y_R}}(k_q, k_p)^* + \overline{\overline{Y_S}}(k_q, k_p)^* \right]^{-1} \vec{V}_{sf}(k_q)^* \right\}^T \left\{ \overline{\overline{Y_R}}(k_q, k_p) \left[\overline{\overline{Y_R}}(k_q, k_p) + \overline{\overline{Y_S}}(k_q, k_p) \right]^{-1} \vec{V}_{sf}(k_q) \right\} \right\} \qquad (3.13)$$

with

$$\frac{1}{C} \overline{\overline{Y}}_{pq}^{-1} \vec{v}_p = \vec{F}_q \qquad (3.14)$$

For high frequencies, moreover, the cross-order terms asymptotically vanishes [2] so (3.13) simplifies to

$$W = \frac{1}{2} C^2 \, \mathrm{Re} \left\{ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} V_{sf\,q}^* \left[Y_{R\,qq}^* + Y_{S\,qq}^* \right]^{-1} \hat{Y}_{qp} V_{sf\,p} \left[Y_{R\,pp} + Y_{S\,pp} \right]^{-1} \right\} \qquad \text{...(3.15)}$$

with $F_q = \frac{1}{C} V_{sf\,q} \left[Y_{Rqq} + Y_{Sqq} \right]^{-1}$ whereby no inversion of the mobility matrices is necessary.

4. MEASUREMENTS

As seen, the computations and measurements for the complete matrix formulation can become cumbersome and simplifications are necessary and desirable.

The aim of this experiment is to examine whether the transmitted power of the servo steering pump can be described in terms of one or two components of motion, and to establish where the *frequency limit* of such a simplification falls.

The test rig consists of a servo steering pump (including a 12V DC motor) mounted on to a plywood plate.
In the preliminary investigation, the velocities were measured on the surface of the springs vulcanized to the pump. Of course, the high mobility of the source surface increases the likelihood that the accelerometer interacts significantly with the source so as to establish an additional mass spring system.
The main investigation has been carried out with the springs disconnected from the pump. The connection points are situated approximately on a circle around the pump at an angle of 45° to the plywood plate.
Two components of motion were obtained, the axial parallel to the stands and the radial component perpendicular to the axial component.

In order to predict the sound power transmission the magnitude and the phase of the *mobilities of the source and the receiver* must be known.
To deduce the mobility of the source, was measured the velocity and the force at the contact points. In the preliminary investigation the high source mobility introduces experimental complications in that the transducers interact with the source constituting a resonant system. This could be avoided by separating the springs from the pump.

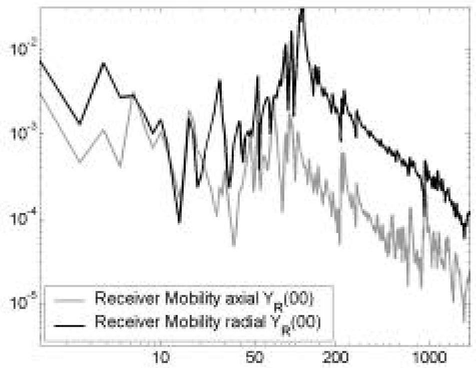

Figure 2: Magnitude of mobility of the receiving structure

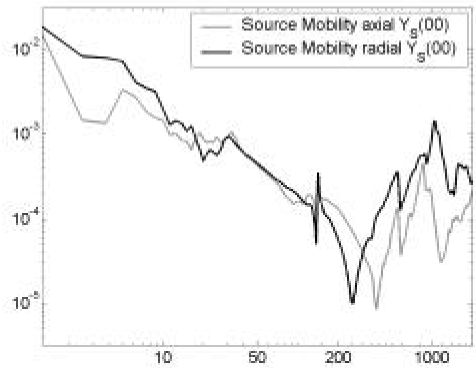

Figure 3: Magnitude of mobility of the source

In order to obtain the activity of the source, herein represented by the *free velocity*, the velocity of the unmounted source was measured. The pump was freely suspended by soft cords to allow the pump to vibrate freely.
The measurements shows, that only at the rotational frequency of the pump (45 Hz) the zero order motion dominates the free velocity of the source. High differences between closely located measurement points were observed, and further investigations are necessary to clarify this anomaly.
The *vibration power* was calculated from 20 accelerator recordings at different, randomly chosen points and additionally from the axial component of force and velocity obtained at the three mount points. These later are regarded as the reference measurements.
The comparison of the predicted and measured power shows for the low frequency acceptable agreement. In this frequency range the measurement arrangement employed in this study produces good results.

5. CONCLUDING REMARKS

The power transmission from source to receiver can be predicted from the velocity of the free source at the contact points and the mobilities of source and receiver. This procedure provides a general way to estimate the power transmission without idealisation of neither the source nor the interface.
The experimental work shows that the simplification of the experimental design produces valid predictions of sound transmission at low frequencies. The experimental effort increase rapidly

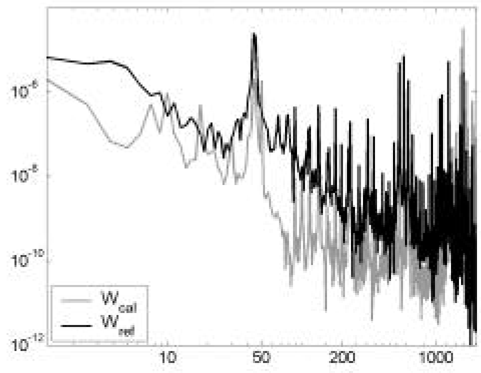

Figure 4: Comparison of the reference measurement - black - (pump mounted on the plywood plate) with the power - grey -, calculated from the mobilities of source and receiver and the free velocity

with the obtained contact points and components of motion, which emphasizes the necessity of simplification.

The study shows the possibility to describe complex sources and receiver in terms of interface mobility. The three-dimensional motion of a non-idealized source complicates the characterization of internal activity whereby this precision predict the preciseness of the calculation of the power.

Above 50 Hz the wavelength of the chipboard plate is close to the dimension of the radius interface, so the high order modes are no longer more small compared with those of the zero order. A further series of experiments is required to demonstrate the applicability of this concept at high frequencies.

6. ACKNOWLEDGEMENTS

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7. BIBLIOGRAPHY

[1] J. Liang and B.A.T. Petersson. Vibrational power transmission in structures built by dynamically mismatched substructures. In 17th ICA Proceedings, Rome, page 3A.04.04, 2001.

[2] B.A.T. Petersson. Interface mobility -theory and applications. DAGA 2001 proceedings.

[3] T. Kihlman. Prediction of structure-borne sound from source and foundation data. Inter Noise, volume 2, pages 437--440, 82.

[4] J. M. Mondot and B. Petersson. Characterization of structur-borne sound sources: The source descriptor and the coupling function. Journal of Sound and Vibration, 114(3):507--518, 1987.

[5] R.J. Pinnington. Vibrational power transmission from a short source beam to a long finite receiver beam via a vibration isolator. In Recent Advances in Structural Dynamics, April 1984.

[6] R. J. Pinnington and Pearce D. C. R. Multipole expansion of the vibration transmission between a source and a receiver. Journal of Sound and Vibration, 142(3):461--479, 1990.

[7] B. Petersson. Geometrical and spatial effects on effective mobilities of annular interfaces. Journal of Sound and Vibration}, 202(4):511--537, 1997.

[8] B.A.T. Petersson. Efficiency of annulary distributed moments and force excitation regarding structural acoustic power transmission to plate-like structures. Journal of Sound and Vibration, 176:625--639, 1994.

[9] L. Cremer and M. Heckl. Körperschall. Springer, Berlin, 1996.

[10] B.A.T. Petersson. An experimental technique for dynamic characteristics at large structural interfaces. In Sixth International Congress on Sound and Vibration}, pages 2175--2182, 1999.