APPLICATION OF EQUIVALENT PRESENTATIONS FOR THE ANALYSIS OF SOUND TRANSMISSION LOSS OF PANELS WITH RESONANT SYSTEMS

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ABSTRACT

The effects of increasing the sound transmission loss of panels with the help of resonant systems on the basis of equivalent presentations are analyzed. The universal expression for the sound transmission loss of panels with resonators of different type is presented. It includes only the parameters common for all the system types (total mass, compressibility, high quality, characteristic frequencies). This expression can be directly used for comparing the efficiency of different resonant system types placed on the panel for determining their optimum parameters.

1. INTRODUCTION

The problem of designing partitions with high sound transmission loss at low frequencies is not solved as yet in many respects. It becomes especially urgent when severe limitations relations relating to mass and thickness are set to the partition, since the sound transmission loss at low frequencies is limited from above by the mass law, as a rule. In the case when the spectrum of the noise to be isolated is dominated by a narrow band or a discrete component, this problem can be solved with the use of partitions on which the system of resonators tuned to the necessary frequency is set. To design such partitions, the resonators of different types can be used.

In the authors’ works [1,2,3] the sound transmission loss of partitions with different types of resonance systems is investigated: with Helmholtz resonators, dynamic vibro-absorbers, acoustic compensators (figure 1). It is shown that a sharp increase of sound transmission loss with the use of these systems is determined by one of three physical phenomena: soft reflection when the input impedance of the panel sharply decreases; hard reflection when the input impedance sharply increases; compensation when the input impedance takes almost pure imaginary value. According to this, any resonance system used for increasing the sound transmission loss can be referred to one of three types: soft reflectors (SR), hard reflectors (HR), acoustic compensators (AC).

In the works indicated the principal attention was paid to resonance systems set on the one side of a solid panel. Such systems will be called single-sided systems. This work pays the main attention to the resonance systems which are set into the holes on panels. The inertial body of such resonators interacts simultaneously with media on each side of the panel. Such systems will be called double-sided resonance systems. To analyze the sound transmission loss of panels with double-sided resonance systems, the method of equivalent presentations will be used, the essence of which is in finding the acoustically equivalent systems. This method application is rather convenient, since it permits combining the systems of different scheme resonators in one type and using unified relations for acoustic property evaluations to analyze them.

2. EQUIVALENT PRESENTATIONS

Consider the system of double-sided compressible resonators shown in figure 2. The compressible resonators are those, oscillations of which are accompanied by their volume variation. The oscillation axis of resonators can be inclined at some definite angle. Such a system is characterized by the following parameters: total surface mass of the system (M); resonator mass per area unit (m); rigidity related to an area unit (k), which connects them and which consists of the air volume rigidity and of fastening rigidity; high-quality (Q); relative areas of interaction ($\mathbf{s}_1, \mathbf{s}_2$); inclination angle of the oscillation axis ($\mathbf{a}$) for the case of inclined resonators.

Each of the resonance systems considered in the work can be treated as a particular case of double-sided compressible resonators (figure 2). Single-sided compensators (figure 1) are a particular case of double-sided resonators when one of the interaction areas is equal to zero ($\mathbf{s}_1 = 0$ or $\mathbf{s}_2 = 0$). Helmholtz resonators can be treated as a particular case of single-sided compensators with a small mass of inertial bodies of resonators. The panel with dynamic vibro-absorbers is formed at zero interaction areas ($\mathbf{s}_1 = \mathbf{s}_2 = 0$). Non-compressible double-sided resonators (figure 3) can be also treated as a limiting case of compressible ones when the interaction areas are equal ($\mathbf{s}_1 = \mathbf{s}_2$). Therefore the relations for double-sided compressible resonators will be valid for all the systems considered.

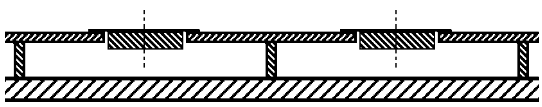

Fig. 1. Acoustic compensators.

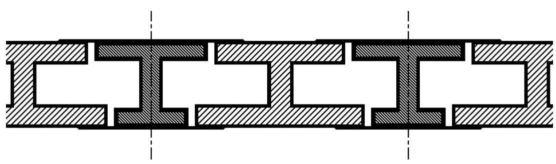

Fig. 2. Double-sided compressible resonators (acoustic compensators).

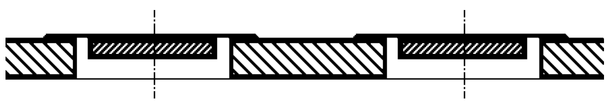

Fig. 3. Double-sided non-compressible resonators (hard reflectors).

To analyze such systems we shall use the simplest mathematical model of normally incident sound waves passing through a uniform infinite purely inertial panel with a regular system of identical resonators, high-quality of which is assumed to be known. The resonator dimensions and the distance between them are small in comparison with the wave length and the system position is given by two coordinates: panel displacement and displacement of inertial bodies of resonators.

The transfer matrix for double-sided compressible resonators (AC2), both straight and inclined, can be written in one of two ways:

$$A_{AC2} = \begin{pmatrix} 1 - M_2\omega^2/k_h\Omega_t & i\omega M\,\Omega_{12}/\Omega_t \\ i\omega/k_h\Omega_t & 1 - M_1\omega^2/k_h\Omega_t \end{pmatrix} = \begin{pmatrix} 1 & i\omega M_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i\omega/k_h\Omega_t & 1 \end{pmatrix}\begin{pmatrix} 1 & i\omega M_1 \\ 0 & 1 \end{pmatrix} \tag{1}$$

$$\Omega_t = 1 - (\omega/\omega_t)^2 + i\omega/\omega_t Q_t\,; \qquad \begin{cases} M_1 + M_2 = M\,; \\ M_1 M_2 = k_h M(\omega_{12}^{-2} - \omega_t^{-2}). \end{cases}$$
$$\Omega_{12} = 1 - (\omega/\omega_{12})^2 + i\omega/\omega_t Q_t\,;$$

It is seen that the transfer matrix with the accuracy up to M_1 and M_2 replacement (this corresponds to turning the system over) is expressed through five parameters: M is the total surface mass of the partition; k_h is the static rigidity of the system determined by its volume variation at pressure variation; ω_t is the tuning frequency of the system at which the maximum sound transmission loss is observed at rather large high-quality; ω_{12} is the resonance frequency of mass-elasticity-mass at which a gap in the sound transmission loss is observed; $Q_t = Q\sqrt{k/m}/\omega_t$ is the equivalent high-quality.

For straight double-sided compressible resonators the values included in (1) are determined as follows:

$$k_h = \frac{k}{(\sigma_1 - \sigma_2)^2};\ M_1 = \frac{m - M\sigma_2}{\sigma_1 - \sigma_2};\ \omega_t^2 = \frac{k}{M\sigma_1\sigma_2 + m(1 - \sigma_1 - \sigma_2)}\ . \tag{2}$$

The expressions for double-sided compressible inclined resonators coincide with those for the straight ones, if a replacement is made in them:

$$m' = m\cos^2\alpha,\ k' = k\cos^2\alpha,\ \sigma_1' = \sigma_1\cos\alpha,\ \sigma_2' = \sigma_2\cos\alpha\ . \tag{3}$$

Hence the inclined resonators are equivalent to the straight ones, if their parameters are related according to (3), where the prime indicates the parameters of straight resonators. The resonance frequency of mass-elasticity-mass is determined as

$$\omega_{12} = \sqrt{kM/((M - m)m)} \tag{4}$$

for the straight resonators of any kind and as

$$\omega_{12} = \sqrt{kM/((M - m\cos^2\alpha)m\cos^2\alpha)} \tag{5}$$

for the inclined ones.

For single-sided compensators $(\sigma_2 = 0)$ positioned on the side 1, expressions (2) are of the following kind:

$$k_h = k/\sigma^2;\ M_1 = m/\sigma;\ \omega_t^2 = k/(m(1-\sigma)) \quad \text{- for straight compensators (6)}$$

$$M_1 = m\cos\alpha/\sigma;\ \omega_t^2 = k/(m(1 - \sigma\cos\alpha)) \quad \text{- for inclined compensators (7)}$$

For straight acoustic compensators the value of M_1 agrees with the surface mass of material, of which their inertial bodies are made.

For non-compressible double-sided straight or inclined resonators (HR2) ($\sigma_1 = \sigma_2 = \sigma$) and for a panel with dynamic vibration absorbers (DVA) the matrix takes the following identical form:

$$A_{HR2,DVA} = \begin{pmatrix} 1 & i\omega M\,\Omega_{12}/\Omega_t \\ 0 & 1 \end{pmatrix}, \qquad (8)$$

where the tuning frequency of the panel with DVA corresponds to their resonance frequency $\omega_t^{DVA} = \sqrt{k/m}$; for double-sided non-compressible straight resonators it is determined from the expression

$$\omega_t^2 = k\left(M\sigma^2 + m(1-2\sigma)\right)^{-1} \qquad (9)$$

and for those inclined it is found from the expression

$$\omega_t^2 = k\left(M\sigma^2 + m(1-2\sigma\cos\alpha)\right)^{-1}. \qquad (10).$$

For the panel with Helmholtz resonators ($\sigma_2 = 0, \sigma_1 << 1$) the direct interaction of which with the panel is neglected, the tuning frequency of the system is the resonance frequency and the transfer matrix for it is written as follows:

$$A_{SR} = \begin{pmatrix} 1 & i\omega M \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ i\omega/k_h\Omega_t & 1 \end{pmatrix} k_h = \rho c^2/h \quad . \qquad (11)$$

where h is the averaged thickness of air cavities of resonators. Further it will be convenient to use a dimensionless combination $\overline{h} = h\omega_t/c$, i.e. the wave thickness. Similarly to Helmholtz resonators, we shall use the equivalent wave thickness $\overline{h}_e = \rho c\omega_t/k_h$ for other compressible resonators.

Thus, the transfer matrix of any of the systems considered can be expressed through the same parameters $(M, k_h, \omega_t, \omega_{12,} Q_t)$. Therefore, if the values of these parameters for two systems are identical, they are equivalent at the correct selection of orientation, i.e. they are not discernible from the acoustic standpoint.

Acoustically equivalent at the respective selection of parameters are the panels with Helmholtz resonators and the panels with compressible resonators turned to 90^o; the panels with DVA and with double-sided non-compressible straight or inclined resonators; single-sided compensators and double-sided compressible resonators which both can be straight or inclined. It means that any resonance system relates to one of three system classes, namely, to soft reflectors, hard reflectors or compensators. Resonators of one class can be acoustically equivalent. So double-sided non-compressible and compressible resonators may be called double-sided hard reflectors and double-sided acoustic compensators, respectively.

Two systems are equivalent with accuracy to orientation, if their total mass, tuning frequency and equivalent high-quality coincide. Besides, for the soft reflectors equivalent wave thicknesses must coincide; for hard reflectors the frequency of mass-elasticity-mass resonance must coincide; for compensators the equivalent wave thicknesses and the frequency of mass-elasticity-mass resonance must coincide.

According to the second presentation of compensator transfer matrix in (1), the panel with single- or double-sided compensators is equivalent to the system consisting of two panels with surface masses M_1, M_2 and with Helmholtz resonators between them with the same tuning frequency, equivalent high-quality and equivalent thickness. Note that masses M_1, M_2 can be negative. Therefore it is reasonable to use the equality of these two parameters (M_1, M_2) as a equivalence criterium of compensators. In particular, it is seen from such an equivalent presentation for compensators that a compensator is equivalent to a panel with soft reflectors on condition that $M_1 = M$ or $M_2 = M$.

3. SOUND TRANSMISSION LOSS OF PANELS WITH RESONANCE SYSTEMS

The sound transmission loss (TL) of panels with resonance systems of any type is determined with the following expression:

$$TL = 20\lg\left|\frac{1}{2}\left(\frac{a_{12}}{\rho c} + a_{11} + a_{22} + a_{21}\rho c\right)\right| = 20\lg\left|1 + i\frac{\left(\overline{h} + M\omega(\Omega_{12} + i\overline{h})/\rho c\right)}{2\Omega_c}\right| . \quad (12)$$

At the tuning frequency TL is approximately expressed as follows:

$$TL(\omega_t) \approx 20\lg(\overline{M}/2) + 20\lg\left|\left(1 - \omega_t^2/\omega_{12}^2 + i\overline{h}\right)Q_t\right|,$$
$$\overline{M} = M\omega_t/\rho c,\ Q_t >> 1 . \quad (13)$$

Introduce efficiency factor Θ that can be used for an approximated expression of panel with resonators TL at tuning frequency in the following form:

$$TL(\omega_t) \approx 20\lg(\overline{M}/2) + 20\lg(\Theta Q_t),\ Q_t >> 1. \quad (14)$$

It follows from (13,14) that the efficiency factor Θ for the resonators of any type is found in the following way:

$$\Theta = \left|1 - \omega_t^2/\omega_{12}^2 + i\overline{h}\right| . \quad (15)$$

In particular, for soft reflectors $\omega_t = \omega_{12}$ and $\Theta_{SR} = \overline{h}$, for hard reflectors $\overline{h} = 0$ and $\Theta_{HR,HR2} = \left|1 - \omega_t^2/\omega_{12}^2\right|$. For hard reflectors the efficiency factor can be also expressed with convenience through a mass of equivalent DVA m' as $\Theta_{HR,HR2} = m'/M$. For double-sided hard reflectors the mass of equivalent DVA is determined by equality $m' = (m - M\sigma)^2 / \left(M\sigma^2 + m(1 - 2\sigma)\right)$. For compensators the efficiency factor is convenient to express through panel masses M_1, M_2 in the equivalent presentation:

$$\Theta_{AC,AC2} = \overline{h}\left|\overline{M}_1\overline{M}_2/\overline{M} - i\right| . \quad (16)$$

Consider the dependency of efficiency factor for different types of resonators according to their mass. Figure 4 shows the efficiency factor dependency of hard reflectors of different types on the mass part accounted for the resonators (m/M). Note that for hard reflectors it is independent of frequency and does not exceed unity. For straight double-sided hard reflectors (HR2) the efficiency factor is transformed to zero, when the surface masses of the panel material and of the resonators coincide ($m = M\sigma$) and it approaches unity, when the resonator mass or the panel mass tends to zero. The Figure indicates that with the use of the resonator oscillation axis revolution (HR2-90, HR2-180), the efficiency factor value can be increased at unvariable resonator mass.

Figure 5 shows the efficiency factor dependency for compensators and soft reflectors. This factor is the minimum corresponding to that of soft reflectors with the same equivalent thickness, when the tuning frequency agrees with the frequency ω_{12}, and when $M_1 = 0$ or $M_2 = 0$. For reversed single-sided compensators (AC-180) the efficiency factor is substantially higher than that for straight compensators (AC). At small masses of resonators double-sided compensators (AC2) are characterized by higher efficiency factor. The compensator efficiency factor value corresponding to its local maximum is independent of the compensator scheme and is $\Theta = \overline{h}\left|\overline{M}/4 - i\right|$, since the compensators in this case are equivalent to one system of two panels with equal masses ($M_1 = M_2 = M/2$) and Helmholtz resonators.

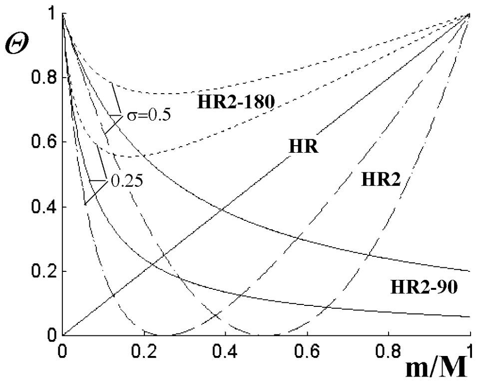

Fig. 4. Efficiency factor of hard reflectors: panel with DVA (HR); straight double-sided hard reflector (HR2); rotated through 90^o (HR2-90); turned over (HR2-180).

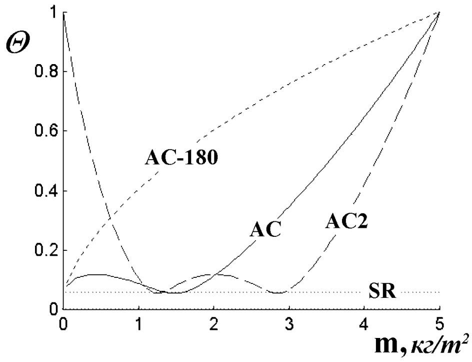

Fig. 5. Efficiency factor for acoustic compensators: single-sided (AC); reversed single-sided (AC-180); double-sided (AC2), and for soft reflectors (SR).

M=5êã/ì^2, h=0.03ì , f_t=100Ãö, s_1=0.25,

for AC2 s_2 varied from 0.5 to 0.6.

CONCLUSION

On the basis of equivalent presentations a wide category of resonance systems with one freedom degree which can be used for increasing the sound transmission loss of panels is examined. The acoustic equivalence of panels with double-sided non-compressible resonators and panels with DVA, of panels with acoustic compensators and with double-sided compressible resonators is established. The universal relation for evaluation of the effect of setting the resonators on a single panel is derived.

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