REDUCTION OF STRUCTURE BORNE SOUND BY NUMERICAL OPTIMIZATION

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ABSTRACT

A method for optimizing structures numerically with respect to structure borne sound is presented. The finite element method (FEM) is used to calculate the surface velocities of a vibrating structure. Powell's COBYLA algorithm is then used to modify the geometry of the structure (*i.e.* the wall thickness) in such a way that the level of structure borne sound is reduced whereas the mass of the structure does not increase. A new FEM input file is generated and the whole process is repeated iteratively until a stop criterion is met. Improvements of more than 6 dB are achieved.

INTRODUCTION

For many reasons (environmental aspects, health, legal regulations, *etc.*) it is desirable to reduce the sound power emitted by a machine structure (*e.g.* gear box, engine block, *etc.*). One way to achieve this is to reduce the so-called level of structure borne sound by means of geometric modifications. From the equation

$$L_{\mathrm{P}}(f) = L_{\mathrm{F}}(f) + L_{\mathrm{Sh}_t^2}(f) + L_s(f) \quad , \tag{1}$$

it can be seen [6] that the level of radiated sound power $L_{\mathrm{P}}(f)$ at frequency f can be written as the sum of the level of the exciting force $L_{\mathrm{F}}(f)$, the level of the radiation efficiency $L_s(f)$, and the level of structure borne sound $L_{\mathrm{Sh}_t^2}(f)$ defined in equation (3). Therefore, it can be assumed that a significant reduction of the level of structure borne sound can lead to a significant reduction in the radiated sound power level as well.

The paper presented here is a continuation of Hibinger's work [4]. He optimizes the level of structure borne sound by applying sequential linear programming (SLP) to three-dimensional model structures consisting of rectangular plates. Differences and improvements include the usage of Powell's COBYLA algorithm, the use of three-dimensional solid elements for the FE models which offer more flexibilty, and the implementation of spline functions to reduce the number of design variables.

NUMERICAL OPTIMIZATION

The COBYLA Algorithm

The COBYLA algorithm (**C**onstraint **O**ptimization **BY** **L**inear **A**pproximation) by Powell [7,8] is a sequential trust-region algorithm which tries to maintain a regular-shaped simplex over the iterations. A sequence of iterations is performed with a constant trust-region radius. Only if the computed objective function reduction is much less than the predicted reduction the trust-region radius is reduced.

COBYLA uses an estimation of gradients by linear interpolation of the objective function. Therefore, no derivatives of the objective function have to be calculated which reduces computation time.

The objective function $F(\boldsymbol{x})$ which is a function of a vector of design variables $\boldsymbol{x}$ is to be minimized subject to constraints $c_i(\boldsymbol{x}) \geq 0$ $(i = 1,2,\ldots,m)$. It is substituted by a merit function [7]

$$\Phi(\boldsymbol{x}) = F(\boldsymbol{x}) + m\left[\max\left\{-c_i(\boldsymbol{x}) : i = 1,2,\ldots,m\right\}\right]_+, \quad \boldsymbol{x} \in \Re^n \quad , \qquad (2)$$

where m is a parameter which is adjusted automatically. The subscript + denotes the expression in square brackets is replaced by zero if its value is negative. Therefore, $\Phi(\boldsymbol{x}) = F(\boldsymbol{x})$ holds whenever $\boldsymbol{x}$ is feasible. This is an elegant way to incorporate the constraints into the objective function.

The Optimization Procedure

After creating an FE model of the original structure a finite element analysis is carried out to calculate the surface velocities evoked by a harmonic excitation force. The value of the objective function prior to the start of the iterations is then determined in a subprogram called OFAC (**O**bjective **F**unction **A**nd **C**onstraints). After that, the COBYLA algorithm modifies the geometry of the structure (*i.e.* its wall thickness) iteratively in such a way that the mean level of structure borne sound in a frequency range (see equation (4)) is reduced while the constraints remain inviolate.

After a new finite element analysis of the modified FE model the value of the objective function (*i.e.* the mean level of structure borne sound) and the constraints are computed again. Objective function and constraints are provided for the COBYLA algorithm which in turn outputs a new modified structure. This process is repeated iteratively until a stop criterion is met.

It is important to mention here that the term "optimization" in this context is not meant in its true sense. For a rather complex optimization problem like this one it would be extremely difficult or maybe even impossible to prove mathematically that a global optimum was found instead of just a local one. Therefore, we speak of an "optimized structure" if we are able to reduce the value of the objective function considerably (*e.g.* by at least 50%). Because of this it would be more precise to speak of an "improved structure".

The Finite Element Model

The pre- and post-processing software MSC.Patran is used to model the original structure. Figure 1 depicts a rectangular plate (350×233.33×4 mm^3) made of steel (density $\boldsymbol{r} = 7850$ kg/m^3, elastic modulus $E = 2.0405 \cdot 10^{11}$ N/m^2, Poison's ratio $\boldsymbol{n} = 0.3$). A harmonic force F excites the plate at the location shown in figure 1. The FE-software ABAQUS is used for the FEM analysis. The plate shown in figure 1 consists of 96 three-dimensional 20 node solid brick elements of the type C3D20 and is simply supported along its edges. Damping is assumed to be frequency independent with a constant modal damping coefficient of 0.4%.

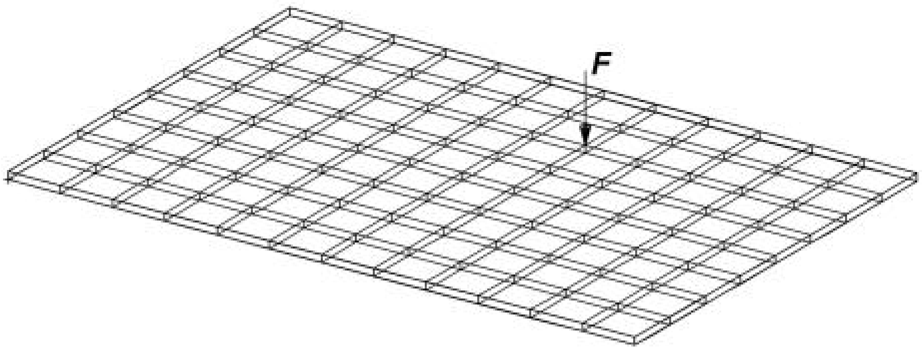

Figure 1: Original FEM model of the plate to be optimized

Objective Function And Constraints

The objective function for the optimization presented in this paper is the mean level of structure borne sound within a frequency range. From the effective surface velocities $\tilde{v}_i(f)$ determined by the FE analysis, the mean squared transmission admittance $h_t^2(f)$ and the level of structure borne sound $L_{\mathrm{Sh_t^2}}(f)$, respectively, are calculated by [6]

$$h_t^2(f) = \frac{\frac{1}{n}\sum_{i=1}^{n}\tilde{v}_i^{\,2}(f)}{\tilde{F}^2(f)} \quad \text{and} \quad L_{\mathrm{Sh_t^2}}(f) = 10\,\lg\frac{S\,h_t^2(f)}{S_0\,h_{t\,0}^2}\,\mathrm{dB}\ , \tag{3}$$

where n is the number of nodes at the sound radiating surface, $\tilde{F}(f)$ is the effective excitation force, S is the radiating surface, and $S_0\,h_{t\,0}^2 = 25\cdot 10^{-16}\,\mathrm{m}^4/(\mathrm{s}^2\mathrm{N}^2)$ is a standardized reference value.

$L_{\mathrm{Sh_t^2}}(f)$ is the spectrum of the level of structure borne sound which is a function of frequency f. For an optimization procedure, however, it is necessary to have a scalar objective function $F(\boldsymbol{x})$ which depends on the vector of design variables $\boldsymbol{x}$ only. Therefore, a mean level of structure borne sound over a frequency range from $f_{\min}$ to $f_{\max}$

$$\overline{L_{\mathrm{Sh_t^2}}} = 10\,\lg\frac{\int_{f_{\min}}^{f_{\max}}\left[S\,h_t^2(f)/S_0\,h_{t\,0}^2\right]\mathrm{d}f}{f_{\max}-f_{\min}}\,\mathrm{dB} \tag{4}$$

is calculated by numerical integration [10] to serve as the objective function for the optimization.

The original plate structure has a constant wall thickness of $t_0 = 4$ mm. $t_{\min} = 1$ mm and $t_{\max} = 10$ mm are chosen as the lower and upper limit, respectively, for the wall thickness of the optimized structure. Furthermore, the upper limit for the total mass of the optimized structure is chosen to be equal to the total mass of the original structure ($m_{\max} = m_0 = 2.564$ kg).

Employing Spline Functions

The wall thicknesses at the surface nodes are the design variables $\boldsymbol{x}$. From figure 1 it can be seen, that even for this rectangular plate the number of design variables is quite large which increases computation time. Therefore, it is desirable to somehow reduce the number of design variables.

This can be achieved by using spline functions to modulate the wall thickness distribution over the surface of the structure [9]. In this way it is not necessary to fit the spline surface to the surface of the FE model prior to the beginning of the iterations which would be another (time consuming) optimization problem by itself. The FE discretization and the spline discretization are completely uncoupled. Therefore, the spline formulation can be changed any time without having to fit the spline surface to the FE surface anew which is another advantage.

Prior to the start of the iterations the thickness distribution of the original structure $d_0(x,y)$ is determined. The wall thickness at the grid point (x,y) is then modified during the iteration process by *e.g.*

$$d_i(x,y) = d_0(x,y) \cdot 5^{S(x,y)} \quad , \tag{5}$$

where $S(x,y)$ represents the value of a spline or polynomial function at the respective grid point [9]. In contrast to the approach presented in an earlier paper [2] where 3D Hermite spline functions are used, 3D bicubic spline functions are implemented here. Figure 2 depicts an example of a surface created by a bicubic spline function consisting of 7x5 key points [1,3,5].

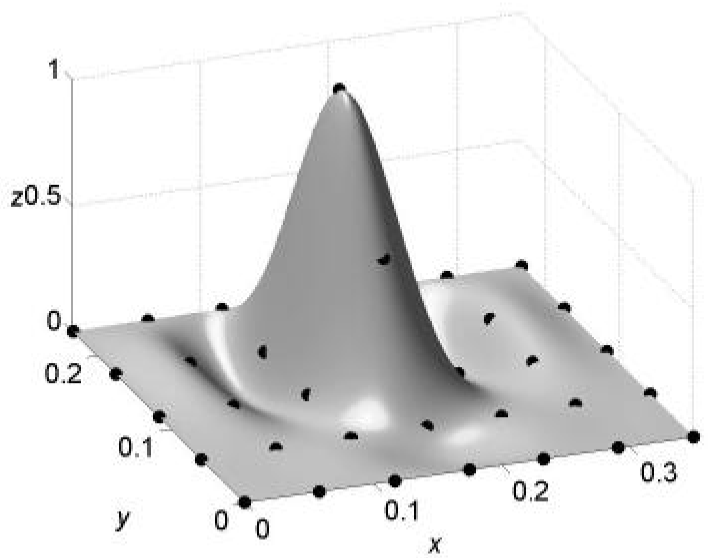

Figure 2: 3D bicubic spline surface (7x5 key points denoted by dots)

NUMERICAL RESULTS

An IBM RS/6000 43P-140 workstation with 384 MB RAM and 233 MHz is used for the optimization calculations. Self-made Fortran routines and UNIX shell scripts are used besides the commercial software mentioned above.

An optimization result for the rectangular plate depicted in figure 1 can be seen in figure 3. Obviously, the algorithm tries to increase the wall thickness of the plate at the location of the excitation force, therefore increasing the input impedance. The wall thickness distribution can also be interpreted as a stiffening rib across the plate reducing the level of structure borne sound at the first eigenmode. The wall thickness along the edges of the plate is held constant during the optimization process.

Figure 4 shows a plot of the number of iterations versus the mean level of structure borne sound. COBYLA varies the z co-ordinates of the 35 key points of the 3D bicubic spline function (design variables $\boldsymbol{x}$) mentioned above. The circles denote feasible results, *i.e.* the objective function is reduced without violating any of the imposed constraints. The mean level of structure borne sound is reduced by more than 6 dB from 83.8 dB to 77.6 dB within a frequency range from 0 to 3000 Hz (see figure 5).

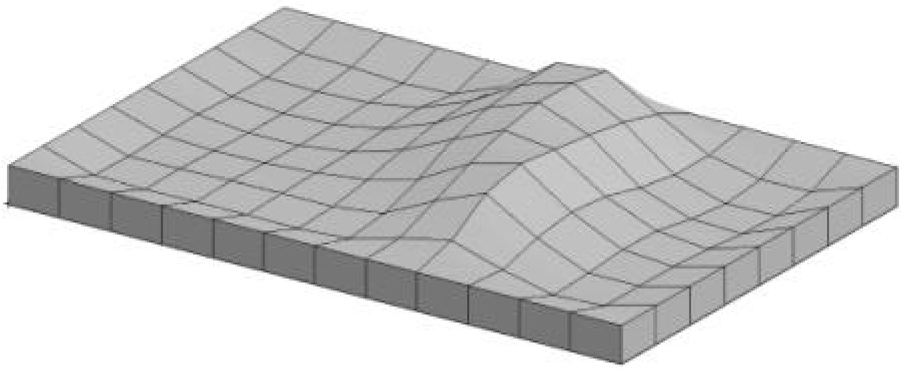

Figure 3: Geometry of the optimized plate (scaled by a factor of 5 in *z*-direction)

The stop criterion is chosen to be a relative change of less than 0.001% between the objective function values of two consecutive feasible results. This criterion is met after 363 iterations (see figure 4). With the bicubic spline formulation implemented here it takes 500 iterations to reach the same mean level of structure borne sound (*i.e.* 77.2 dB) that is reached after 447 iterations using the Hermite spline functions in [2]. After 447 iterations (the number it takes to reach the same stop criterion as in [2]) the mean level of structure borne sound is reduced to 77.3 dB.

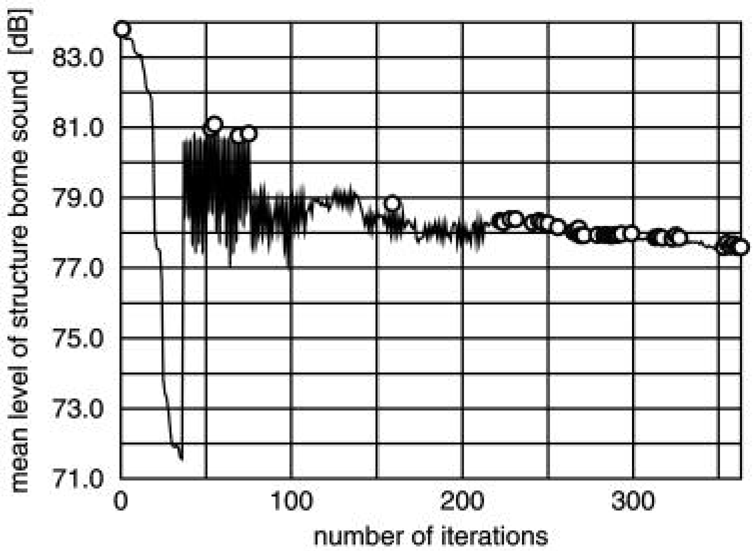

Figure 4: The course of the optimization process (circles denote feasible results)

The spectra of the level of structure borne sound of the original and the optimized plate, respectively, are depicted in figure 5. Due to the optimization not only is the mean level reduced but also the peak level at the first natural frequency is reduced from 105.8 dB to 99.8 dB. The first natural frequency is shifted from 257.4 Hz to 327.7 Hz so that the quasi-static frequency range with its low level of structure borne sound is advantageously extended.

SUMMARY

A simple three-dimensional structure, *i.e.* a simply supported rectangular plate is optimized with respect to the level of structure borne sound using Powell's COBYLA algorithm. Three-dimensional bicubic spline functions are utilized to vary the wall thickness of the plate. In this way the number of design variables can be reduced drastically from 77 to 35. The mean level of structure borne sound over a frequency range from 0 to 3000 Hz which serves as the objective function is reduced by more than 6 dB. In comparison with the bicubic spline functions implemented here the Hermite spline functions used in [2] seem to have some advantages concerning the convergence rate of the optimization process.

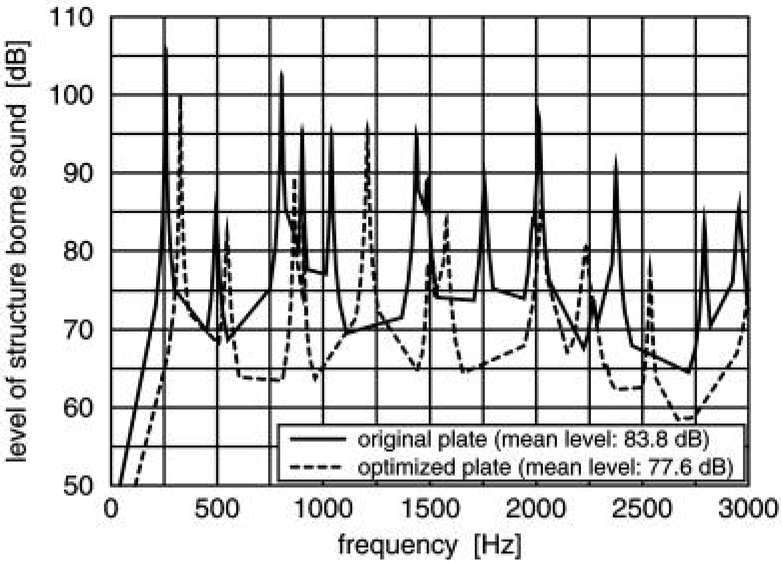

Figure 5: Results of the optimization procedure

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