THE ONE-SIDED DRIVING INTERACTION BETWEEN TWO MONOPOLES - EFFECTIVE AS THE CORE OF THE 3-DIMENSIONAL DFEM SOUND POWER AND SCATTERING DESCRIPTION

Huebner Gerhard
Stuttgart University, ITSM
Pfaffenwaldring 6
D-70550 Stuttgart
Germany
Tel. 049-0711-685-3516
Fax: 049-0711-685-3280
E-mail: Huebner@itsm.uni-stuttgart.de

ABSTRACT
By using the DFEM (Direct Finite Element Method) the sound power radiated by a vibrating body is calculated from its outersurface vibrational and geometrical quantities directly means without determining sound field quantities such as sound pressure or sound intensity as an intermediary step. By this method the sound power radiated by a vibrating plane surface embedding in a large rigid shield can be determined by the „elementary DFEM“ with exactly the same result as given by the Rayleigh's equation followed by a second enveloping surface integration . In this case the vibrating surface is replaced by a net of discrete equivalent monopole souces and its resulting sound power is calculated by the sum of all single monopole powers together with the sum of the interaction sound powers from all pairs of these monopoles . But for true 3 – dimensional sound sources both scattering effects and boundary conditions are to be considered additionally. In this case the elementary DFEM net must be supplemented by a second net of counter monopoles to ensure the boundary condition of a vanishing normal component of the sound velocity caused by the first monopole net . The magnitude of the monopoles of the second net is determined by the one-sided driving interaction between both nets . The paper shows on the one hand the derivation of this interaction effect for the example of two relevant monopoles and gives several illustrations of calculated 3-dimensional „general DFEM“ sound power and DFEM scattering determinations.

INTRODUCTION

More than 20 years ago the Technical Committee 43 „Acoustics“ of ISO decided to characterize the noise emission of machines and equipment by its sound power. This quantity depends on distance and environmental conditions very weekly , contrary to the relevant sound field quantities such as sound pressure and sound intensity. So sound power is a true machinery specific quantity. Consequently in the mean time under the roof of this ISO committee several sound power measurement procedures are developed and issued both based on sound pressure and later on using sound intensity determination (ISO 3740 series, ISO 9614 , part 1, 2 and 3). Furthermore ISO TR 7849 gives an experimental airborne sound power determination by the measurement of the relevant machine outersurface structure borne velocity components.

Regarding the corresponding development in relevant numerical methods two different tracks can be recognized.

The well known **first method** determines the radiated airborne sound power by two steps. At first the Boundary Element Method (BEM) is used to calculate relevant air borne sound field quantities such as sound pressure , velocity or intensity caused by the source in a free field space along several positions on a surface S enveloping the source, where S preferably is situated in the far field . For baffled plane sources the use of the Rayleigh-integral instead of BEM is of significant advantage . Then by the next step the relevant field quantities were integrated by the well known relationships along the enveloping surface S yielding the sound power finally.

The **second method** determines the sound power without any airborne sound field quantity calculations. The so called Direct Finite Element Method (DFEM) determines the air borne sound power directly in one step from the source vibration quantities only together with geometrical source data. This method was issued several years ago both for baffled plane sources [1], [2] and later on was developed, tested and approved for true 3-dimensional sources [2-6].

Furthermore a sound power measurement procedure was developed based on the general DFEM Algorithm [2], [7].

Most of the papers dealing with DFEM are issued in German language. This fact together with the publication at very different places over a time interval of more than one decade may be a reason that at present this method is not very familiar . Therefore this paper intends to summarize and to explaine the main DFEM aspects supplemented by specific references.

BASIC PHILOSOPHY OF THE DFEM ALGORITHM

Introducing the DFEM we regard a system of N different individual sound sources arbitrarily positioned in a free space enveloped by the surface S (fig.1). These sources may have different radiation pattern in respect to directivity and frequency spectra. Based on the principle of superposition the intensity component I_n normal to S effective at each position on S is given by

$$I_n = \overline{\left(\sum_{i=1}^{N} p_i\right)\left(\sum_{i=1}^{N} v_{i,n}\right)}^{\,t} = \sum_{i=1}^{N} \overline{p_i v_{i,n}}^{\,t} + \sum_{i=1}^{N} \sum_{\substack{l=1 \\ i \neq l}}^{N} \overline{p_l v_{i,n}}^{\,t} \qquad (1)$$

where the p_i and $v_{i,n}$ are the sound pressure and normal component of the sound velocity radiated by the i-th source effective in absence of all other N-1 sources . Having integrated I_n over S the total sound power P_Σ of the source system yields

$$P_\Sigma = \sum_{i=1}^{N} P_i + \sum_{i=1}^{N} \sum_{\substack{l=1 \\ i \neq l}}^{N} P_{il} \qquad (2)$$

This means P_Σ is given by the sum of all single sound powers P_i and a sum of all interaction sound powers P_{il} .

Finally the P_i and P_{il} can be expressed by the sound source quantities. As an example this should be shown for the source system consists of monopole sources only. In this case we have *)

$$P_i = \frac{\rho c}{\Omega_i} k^2 (\Delta S_i \tilde{v}_i)^2 \qquad P_{il} = \frac{\rho c}{\Omega_i} k^2 (\Delta S_i \tilde{v}_i)(\Delta S_l \tilde{v}_l) \frac{\sin k d_{il}}{k d_{il}} \cos \varphi_{il} \qquad (3)$$

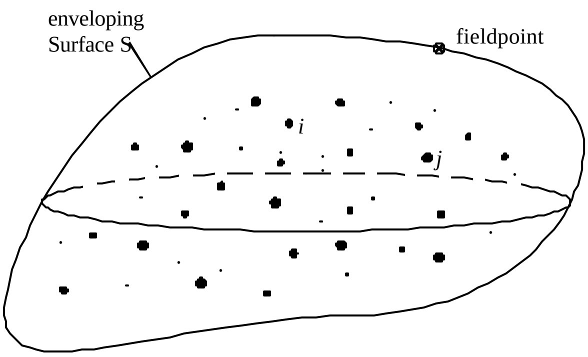

Fig.1 System of individual sources.

with the space angle Ω_i =4π and where ρ c is the characteristic acoustic impedance in air, k=2πf/c the wave number , ΔS_i , ΔS_l the individual monopole surface areas, $\tilde{v}_i, \tilde{v}_l$ the rms values of the monopole velocities, φ_{il} the relative monopole phases and d_{il} the distances between the i-th and the l-th source . Similar description expressing P_i and P_{il} for several different source types e.g. for systems with dipoles , mixed monopole / dipole systems and systems with spherical sources of different orders can be found in [9].

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*) Close to the physics of air borne sound radiation the eqs.(3) can be formulated frequency independently by replacing $k\tilde{v}_i = \frac{2\pi f}{c}\tilde{v}_i$ by $\frac{\tilde{a}_i}{c}$ where $\tilde{a}_i$ is the rms acceleration.

Introducing eqs.(3) into (2) the aim of DFEM is obtained: The radiated airborne sound power is expressed by sound source quantities only such as source frequency f , sourcesvelocities $\tilde{v}_i$, areas ΔS_i and source phase relatioships together with the geometrically determined source distances d_{il} and ρ c characterizing the gas quality in which the sound propagates. Consequently the airborne sound field quantities are not necessary for the DFEM airborne sound power determination . The one step determination requires source and geometrical (d_{il}) quantities only.

DFEM SOUND POWER DESCRIPTION FOR BAFFLED PLANE STRUCTURE BORNE SOURCES -THE „ELEMENTARY DFEM"

For such sources the DFEM replaces its vibrating surface by a net of equivalent monopole sources radiating into the half-space and determines the radiated sound power by eqs.(2) and (3) with $\Omega_i = 2\pi$. For this the sound flows $\tilde{v}_i \Delta S_i$ yields by the discretization of the true $\tilde{v}_n(S)$ distribution.

The evidence of the DFEM determined sound power of baffled plane sources was shown both by comparising with exact analytical solutions of several specific examples (baffled vibrating strips, piston diaph-ragm and plates of different vibration orders [9], [5]) and was basicaly proofed [4], [8] by a derivation starting with the far field sound pressure determination by the Rayleigh's equation continued by a hemisperical surface integration of $\frac{1}{\rho c}\tilde{p}^2$ for a radius R→∞. The DFEM result refered to the Rayleigh-equation derivation-track is exact so long as the chosen density of discretization is large enough related to the relevant air borne wavelength λ respectively to the upper frequency limit f_{max} of interest. Criteria e.g. for a zero order vibrating strip with the length L: the number monopoles N should fulfill $N > (L/\lambda)_{max}$ respectively $N > f_{max} L/c$ (see fig.2).

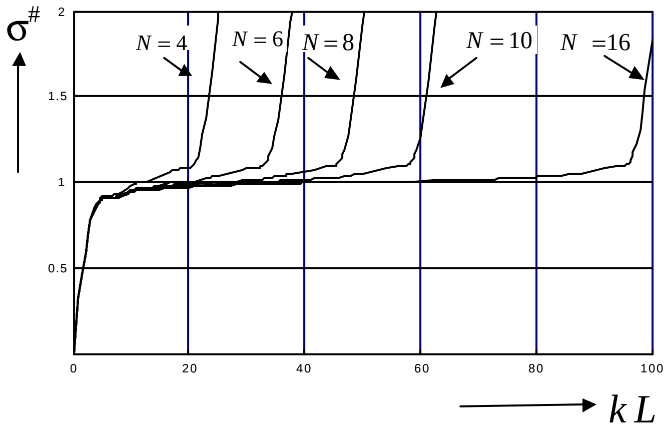

Fig.2 Related radiation efficiency $\sigma^{\#}$ of a zero order vibrating strip determined by DFEM with different discretization N.

DFEM DESRIPTION OF SCATTERING EFFECTS - THE ONE-SIDED DRIVING INTERACTION

Our first equation (3) describes the monopole sound power P radiated under ideal free field conditions. Now we regard the deviation of this sound power, if a scattering body is situated in the vicinity of the source M, as shown by figure 3. Thereby we assume the monopole M with constant sound flow $q_0 = \Delta S \tilde{v}_0$. By the following the solution of this problem is described by using the DFEM. Further relevant details are given by [3].

The free field sound velocity $\mathbf{v}_i$ radiated by the monopole M don't fulfill the rigid body boundary condition by its normal component $v_{n,i}$ at any i-th location on the body's surface S in general (fig.3). Therefore the DFEM locates an additional net of imaginary counter monopoles on S having sound velocities v_i^c to ensure for each i-th position

$$v_{n,i} + v_i^c = 0 \tag{4}$$

Consequently these counter sources have sound flows

$$q_i^c = -v_{n,i}\Delta S_i = -|\mathbf{v}_i|\cos\beta_i\,\Delta S_i \tag{5}$$

where ΔS_i is the portion of the area S refered to the mesh of the i-th counter source and β_i the angle between the free field velosity $\mathbf{v}_i$ and the vector $\mathbf{n}_i$ normal to surface S at the i-th position.

The deviation of sound power P_0 caused by the scattering effect is determined by the **one-**

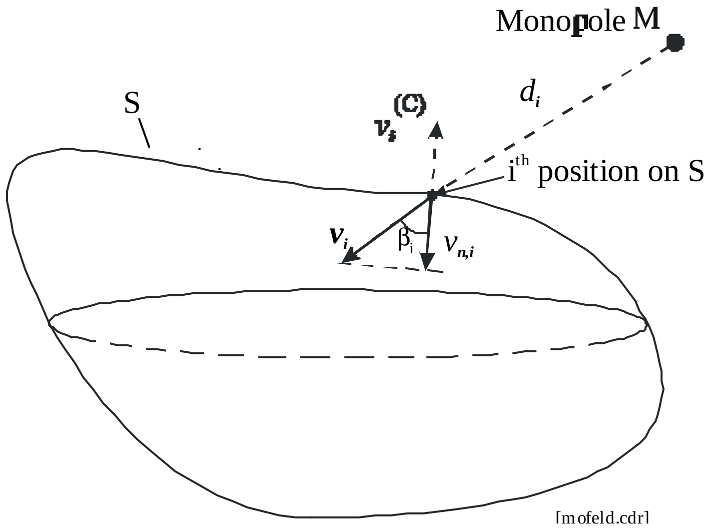

Fig.3 Field situation for a monopole in the vicinity of a rigid body S.

sided interaction between the M monopole sound power P_0 with q_0 and the M_i counter sources characterized by q_i^c. The description of this situation follows the equation (2) and (3) on principle but must be modified in order to consider the specific one-sided effect (details see [2], [9]).

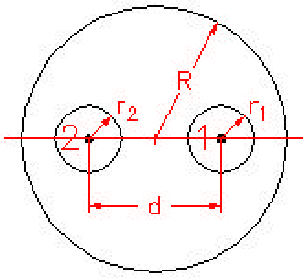

Fig.4 Integration surfaces for a pair of monopoles 1 and 2

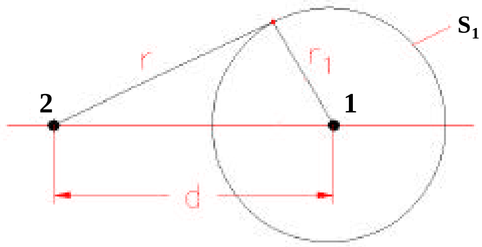

Fig. 5 Integration surface S_1 for the one sided effected influence

For this modification at first we regard a single pair of monopoles being separated by the distance d, having a phase difference ϕ_{12} and constant sound flows q_1 and q_2. The well known total sound power of these two sources is given by

$$P_\Sigma = P_1 + P_2 + 2\sqrt{P_1}\sqrt{P_2}\,\frac{\sin(kd)}{kd}\cos\phi_{12} \tag{6}$$

with symbols as explained before. This sound power P_Σ usually is derived by integrating sound field quantities over a sphere with radius R enveloping both sources (fig.4) . On the other hand the one-sided influence is defined by the change of one of the monopole powers, e.g. of P_1, caused by the presence of the other one . Therefore the relevant sound field quantities must be integrated over the smaller sphere with radius r_1 (fig.4-5). This integration carried out e.g. for the monopol 1 yields to (see [2], [9])

$$P_{1,\Sigma} = P_1 + \sqrt{P_1}\sqrt{P_2}\,\frac{\sin(kd+\phi_{12})}{kd} \tag{7}$$

For the scattering problem as illustrated by fig.3 further development of equation (7) is necessary . At first P_1 is replased by the power P_0 of our primary source M and the P_2 by the i-th counter source power

$$P_i = \frac{\rho c}{\Omega_i} k^2 (\tilde{v}_i^c \Delta S_i)^2 \tag{8}$$

where the counter sound flow $v_i^c \Delta S_i$ is determined by the driven velocity $v_{n,i}$ caused by the primary monopole M which at the i-th position is given by

$$v_i^c = -v_{n,i} = -\frac{1}{4\pi d_i}\tilde{q}_0 k\sqrt{1+\frac{1}{(kd_i)^2}}\, e^{j(\omega t - kd_i + \frac{\pi}{2} - \arctan\frac{1}{kd_i})}\cos\beta_i \tag{9}$$

and finally

$$P_i = \frac{\rho c k^4 \tilde{q}_0^2}{\Omega_i 16 \pi^2 d_i^2}\left(1 + \frac{1}{(kd_i)^2}\right)\cos^2 \beta_i (\Delta S_i)^2 \qquad (10)$$

The phase ϕ_{12} from equation (7) is determined by the retardation caused by wave propagation from M to the i-th point on S (fig.3) with

$$\varphi_{M\,i} = kd_i + \arctan\frac{1}{kd_i} - \frac{\pi}{2} \qquad (11)$$

Introducing P_0 for P_1, P_i according eq.(10) for P_2 and $\phi_{M\,i}$ with eq.(11) for ϕ_{12} into eq.(7) the one-sided driven deviation Δ_i of the sound power P_0 caused by P_i yields

$$\Delta_i = \sqrt{P_0}\sqrt{P_i}\,\frac{\sin(kd_i + \varphi_i)}{kd_i} \qquad (12)$$

The sound power deviation totally caused by the entire scattering body follows

$$\Delta_{tot} = \sum_{i=1}^{N} \Delta_i \qquad (13)$$

One example of this scattering effect is the change of the monopole sound power caused by a rigid rectangular plate with a size limited by $2L_x$ and $2L_y$ being situated in a distance D. As a reference for this example we regard the relevant well known results for a plate unlimited in size ($2L_x$, $2L_y \to \infty$) with

$$\Delta_{tot}^{\infty} = P_0 \frac{\sin(k2D)}{k2D} \qquad (14)$$

As expected for higher frequences, increasing $Dk \sim D/\lambda$ and $(L_x;L_y)k \sim (L_x;L_y)/\lambda$ the DFEM determined Δ_{tot} approximates the Δ_{tot}^{∞} very well and for lower frequencies, means smaller wavelength λ related to the geometric quantities D; Lx; Ly as expected significant differences between these two theoretical deviations can be realized (fig.6 and 7). Further DFEM solutions for other scattering objects, such as circular plates and rigid sphere are given by [2].

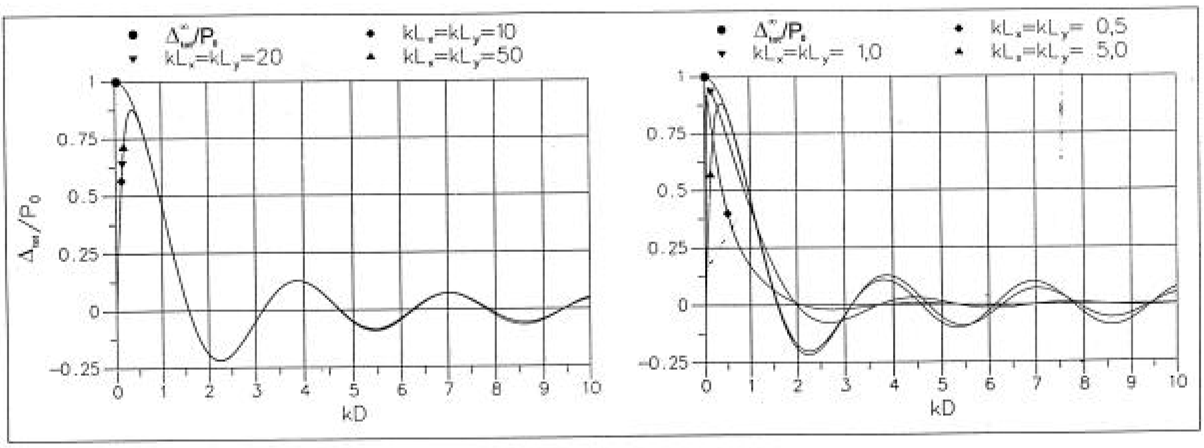

Fig.6 and 7. Related sound power deviation Δ_{tot} /P_0 caused by scattering on a rectanular rigid plane

DFEM FOR SOUND POWER DETERMINATION OF 3-DIMENSIONAL SOURCES

– **THE GENERAL DFEM**

A monopole located in the „north-pole“ of a rigid sphere with the outersurface S generates „free“ sound velocity components $v_{n,i}$ normal to S <u>different</u> of zero if the sphere's surface S is assumed imaginary means being previous to sound (see fig.8). But for increasing the spheres's radius r_0 the component $v_{r,i}$ decreases and vanishes for $r_0 \to \infty$. Means for a plane sources we have $v_{n,i}=0$ automatically. Continuing the discussions of the preceding chapter for the 3-dimensional sources a net of imaginary counter monopoles is necessary to fulfill the boundary conditions on S for the true existing rigid sphere outside the „north-pole“. All earlier derivation of the scatter problem are still usable to determine the counter monopoles when moving our monopole M of fig. 3 close to the surface S ($d_i \rightarrow 0$) and changing the angle faktor $1/4\pi$ into $1/\Omega_i$ where for plane baffled sources counter sources are not necessary. For the Ω_i determination of arbitrarily shaped 3-dimensional sources reference is given to [4], [6].

Finally the general DFEM sound power determination of the 3-dimensional sound sources consists of

- generation of the inputs :

by discretization of the actually given structure borne velocity $v_n(S)$ in respect to the amounts and phases accompanied by fixing the coordinates and angles of ΔS_i positions the primarily monopole net is generated

- realization of the algorithm:

the sum of all single monopole sound powers, the sum of their interaction sound powers both for the primarily net and secondly generated the net of counter sources has to be calculated.

The DFEM Algorithm was checked on a very broad basis

- (1) by comparison the sound power determined by DFEM with results of plane and 3-dimensional sources allowing exact analytical solutions, see references [8], [4], [5], [9].
- (2) by using the DFEM Algorithm for a sound power measurement procedure and checking these results with sound powers determined by a sound intensity measurement procedure (ISO 9614), see references [2], [5-7], [10,11].

For both tests the oral presentation will show several examples. These tests show excelent agreements whereby DFEM avoid any singularity problems.

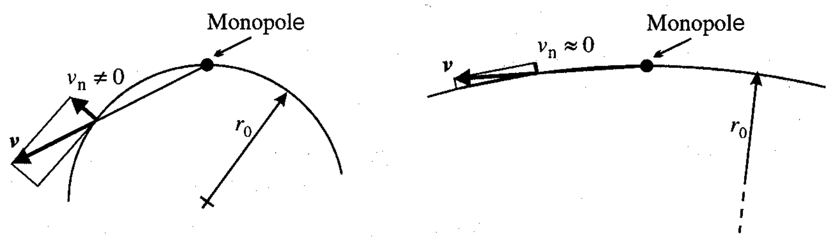

Fig.8 Sound field situation for spherical sound sources different in radius r_0

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