TIME-DOMAIN SIMULATION OF A GUITAR

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ABSTRACT

A time-domain numerical simulation of a guitar is presented. The model includes the transverse motion of strings excited by a force pulse, the flexural motion of the soundboard and the acoustic equations of both the external field and cavity. A specific variational formulation is used for solving the Kirchhoff-Love's dynamic plate problem for an anisotropic inhomogeneous material with free boundary at the soundhole and clamped boundaries along the ribs. In addition, a fictitious domain method is used for solving the fluid-structure interaction.

INTRODUCTION

The scope of the paper is the simulation of the sound pressure field radiated by a guitar in the time domain. Guitar sounds, and guitar music, are essentially made of transients. Therefore, we wish to simulate transient sound field for a better understanding of the time-varying vibroacoustical phenomena involved in a guitar. The method consists in the modeling of the vibratory and acoustical phenomena from the initial pluck to the 3-D radiation. The main features of the continuous model are the following: an idealized plucking force is acting on a 1-D damped string model. The motion of the string is assumed to be perpendicular to the top plate. The top plate is modeled as an orthotropic inhomogeneous damped Kirchhoff-Love plate, with a soundhole, and clamped at the boundaries. The other parts of the guitar (back, neck, sides,…) are assumed to be perfectly rigid.The plate radiates both inside the cavity and in the external free field. The cavity communicates with the external field through the hole.

The spatial part of the coupled partial differential equations which compose the model is solved by the finite element method. This method is based on a variational formulation of the problem. In this procedure we introduce, in particular, the pressure jump λ across the boundary of the instrument, so that it allows the use of a regular mesh for the approximation of the acoustic field, and leads in turn to an efficient numerical scheme in terms of computing time and accuracy. A standard numerical scheme is used for the string equation. The plate equation is solved by modal reconstruction, where the eigenmodes of the soundboard are first calculated by a finite element method. A modal truncation is made, according to the usual spectral content of real instruments. Finite differences in time are used for the string and the acoustic field, whereas exact resolution in time is used for the plate equation. A property of discrete energy decay ensures the stability of this hybrid time resolution.

The modeling of the sound field shows significant new aspects. First, in contrast to previous works on the guitar which use mostly a frequency stationary representation, the time-domain approach yields here the evolution of sound pressure with time inside and outside the instrument. This evolution is of great help for a better understanding of the complexity of sound waves which reach the ears of a listener in a room. It allows, in particular, to reconsider the definition of directivity and efficiency of the instrument, by taking into account the temporal evolution of such quantities. Secondly, there is no discontinuity here between interior and external sound field: the same equations apply to the complete sound field. This is again a major difference with almost all previous works on the guitar, where the cavity is taken into account in terms of Helmholtz resonance and air eigenmodes, and where the reaction of the sound field on the top plate is either neglected or approximated. Traditionnaly, the top plate properties are conveniently summarized by the admittance of particular selected points at the bridge, since that part of the instrument is critical in terms of energy transfer between strings and plate. Therefore, our model has been evaluated by the computation of such admittances, among other criteria.

With regard to previous work on guitars, one can summarize the originality of the present work as follows: It is an alternative to the Boundary Element Method, developed by Richardson *et al*. [3], and to the spatial Fourier method developed by A. Le Pichon *et al.* [6]. The results of the vibratory part can be well compared to the results obtained by Elejabarrieta *et al*., although the present model of the box is highly simplified [8]. As far as the numerical technique is concerned, the present modeling is a direct extension to plates of the timpani model developed by Rhaouti *et al.* [9]. The modeling of the guitar top plate makes also use of results obtained by Chaigne and Lambourg [4], especially with regard to the damping. The present modeling of the pluck is oversimplified compared to the work by Pavlidou [1] which could be incorporated in the model in the future. The modeling of the string uses results obtained on nylon strings by Chaigne [2]. Finally, both the string-plate and plate-cavity coupling confirm and extend previous results obtained by Lambourg *et al* [5].

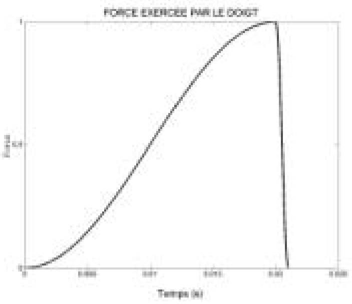
PRESENTATION OF THE MODEL

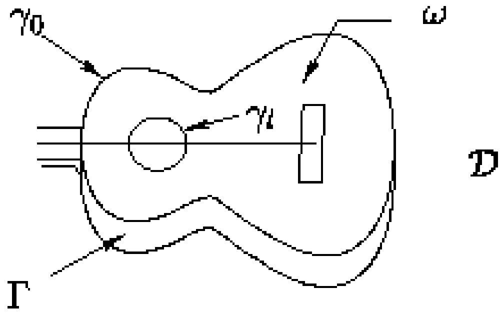
For the sake of simplicity, the motion of the string is assumed to be linear and perpendicular to the top plate and excited by an imposed force pulse (see Equations (1)-(2) and Figure 1). The string is rigidly fixed at the nut and the displacement of the string at the bridge is given by the average displacement of the bridge, which is assumed to be a rigid solid (3). The string is homogeneous with linear density ρ_s and stretched with uniform tension T. The transverse equation of motion also includes fluid damping R_{fc} and viscoelastic damping β. The stiffness of the string is neglected.

$$\rho_s \frac{\partial^2 u_s}{\partial t^2} - T\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial^2 u_s}{\partial x^2} + R_{fc}\frac{\partial u_s}{\partial t} = h(t)g(x) \qquad (1)$$

$$h(t) = \begin{cases} \dfrac{F_m}{2}\left(1-\cos\dfrac{\pi t}{t_1}\right) & t < t_1 \\ \dfrac{F_m}{2}\left(1+\cos\dfrac{\pi (t-t_1)}{t_2}\right), & t \in [t_1, t_2] \\ 0, & t > t_2 \end{cases} \quad \text{and} \quad \int_0^l g(x)dx = 1 \qquad (2)$$

$$u_s(0,t) = 0 \quad \text{and} \quad u_s(l,t) = \int_{S_p} G(x,y)u_p(x,y,t)\,dxdy \quad \text{with} \quad \int_{S_p} G(x,y)dxdy = 1 \qquad (3)$$





Left - Figure 1: Idealized plucking force pulse. Right – Figure 2: Geometry of the guitar. Symbol ω represents S_p, the surface domain of the top_plate.

The transverse displacement u_p of the top-plate (also called soundboard) is governed by the Kirchhoff-Love model for flexural vibrations (4). The material (wood) is assumed to be orthotropic. In Equation (4), ρ_p is the density, δ is the thickness, **C** is the elasticity tensor, η is a viscoelastic coefficient, R_{fp} is a fluid damping coefficient. A global law of damping versus frequency can be derived if η and R_{fp} are kept constant. However, due to numerical reasons, a modal approach has been used, where the damping factor can be adjusted for each mode of the plate, which yields more flexibility for defining the decay times (see the next section). On the right-hand side of Eq. (4), *Fc(t) G* is the force density imparted by the string at the bridge and *[p]* is the pressure jump across the soundboard. The geometry of the top-plate corresponds to the one of a standard classical guitar, including the shape, size and position of the soundhole. The bracing and the bridge are modeled by local inhomogeneities in thickness, stiffness and/or density. The other parts of the guitar body are assumed to be rigid. Due to the flexibility of the model, some of these simplifying assumptions could be easily modified, if necessary. The top plate is assumed to be clamped at its external boundaries γ_0 (5) and free along the edge of the soundhole γ_l (6).

$$\rho_p \delta \frac{\partial^2 u_p}{\partial t^2} + \mathrm{div}\left(\underline{\mathrm{Div}}\,\delta^3 \underline{\underline{\mathbf{C}\varepsilon}}\left(\underline{\nabla}\left(u_p + \eta \frac{\partial u_p}{\partial t}\right)\right)\right) + R_{fp}\frac{\partial u_p}{\partial t} = F_c(t)G - [p]_{S_p} \quad \text{on } S_p \qquad (4)$$

$$u_p = \partial_n u_p = 0 \qquad \text{on} \qquad \gamma_0 \qquad (5)$$

$$\underline{\mathrm{Div}}\,\underline{\underline{M}}.\underline{n} + \partial_s M_{nt} = 0 \quad \text{and} \quad M_{nn} = 0 \quad \text{on } \gamma_l \quad \text{with} \quad \underline{\underline{M}} = \delta^3\,\underline{\underline{\mathbf{C}\varepsilon}}\left(\underline{\nabla} u_p\right) \qquad (6)$$

The acoustic pressure p and velocity V_a are governed by the well-known continuity and Euler equations, in the linear case (7). Keeping both variables have several advantages. From a numerical point of view, it allows to work with first-order partial differential equations. From a physical point of view, it allows to compute energetic quantities such as sound intensity and sound power, which are essential for characterizing the instrument. The damping of the acoustic wave has been neglected. This assumption should be probably reconsidered in the future in order to account for the magnitude of the sound pressure near resonances of the cavity. Equation (8) yields the continuity of the normal velocity at the top-plate. The acoustic velocity is set to zero on the sides and on the back plate (9). The diffraction of the acoustic wave by the neck and head are neglected. The computation of the sound field is made in a small anechoic rectangular box. Absorbing boundary conditions are imposed on the walls [10].

$$\frac{\partial p}{\partial t} = -\, c_a^2\, \rho_a\, \mathrm{div}\underline{V_a} \quad \text{and} \quad \rho_a \frac{\partial \underline{\underline{V_a}}}{\partial t} = -\,\underline{\nabla p} \quad \text{in } \mathcal{D} \qquad (7)$$

$$\underline{V_a}(x,y,0,t).\underline{e_z} = \frac{\partial u_p}{\partial t}(x,y,t), \qquad \forall (x,y) \in S_p, \qquad \forall t > 0 \qquad (8)$$

$$\underline{V_a}(x,y,z,t).\underline{N_\Gamma} = 0 \qquad \forall (x,y,z) \in \Gamma, \qquad \forall t > 0 \qquad (9)$$

NUMERICAL FORMULATION

The proposed modeling of the guitar couples thus a set of string equations, plate equations and acoustic field equations. In order to take into account the complex geometry and the particular boundary conditions, we use the finite element method for the space discretization, which is more adapted to such cases than the finite difference method. The first step of the numerical resolution is thus to rewrite the complete problem into a variational formulation. In addition, this approach is a convenient way to obtain a property of discrete energy decay of the final numerical scheme, which is aimed at ensuring its stability.

One of the main difficulties results from the size of the problem. It is a three-dimensional problem in a domain of infinite size and with a complex geometry. The most efficient method would be to use use a regular mesh for the approximation of the acoustic field, which in fact leads to a finite difference method. However, this choice gives a bad approximation of the shape of the instrument and spurious diffractions appear on this artificial boundary. Another possibility is to use a tetrahedric mesh which fits the geometry of the guitar with a better approximation, but this method is expensive, both in time and memory. Finally, another alternative, which has been introduced by Rhaouti *et al.* for the numerical modeling of a kettledrum, is to use a fictitious domain method [9]. This method preserves the efficiency of the finite difference method and approximates correctly the shape of the guitar. One introduces $\lambda = [p]$ the pressure jump across the boundary of the instrument as new variable, and writes a variational formulation in which the instrument appears only through the Lagrange multiplier λ, so that it allows the use of a regular mesh for the approximation of the acoustic field. For the approximation of λ, we have to mesh the surface of the instrument, which is relatively easy. The price to pay is the inversion of a symmetric matrix of the size of the Lagrange multiplier at each time step. Thus, the scheme is not explicit.

Given a regular mesh of the string, made of small segments, the approximation of the unknown u_s is determined by its values at each node of the mesh (P_1 continuous finite elements). Given a triangular mesh of the top-plate surface S_p, the approximation of the unknown u_p is determined by its values at the vertex and the gravity center of each triangle, and its values at the middle of each edge (P_2 alike-continuous finite element of order 2, see [11]). This particular choice reduces the numerical dispersion effects and leads to a better approximation of the eigenfrequencies of the soundboard. A triangular mesh is given on Γ (see Fig. 3), which coincides with the triangular mesh on S_p, in order to facilitate the computations. The Lagrange multiplier λ is determined by its values at the vertex of each triangle (P_1 continuous finite element). To simulate the free space, the actual computations are restricted to a box Ω of finite size. Given a regular mesh of Ω, made of small cubes, the approximation of p is determined by its values at the middle of each cube (P_0 discontinuous finite element). The approximation of the velocity V_a is determined by its values of its normal component across each face of the cubes (Raviart-Thomas finite element). For the time discretization, a classical centered finite difference scheme is used for the string and for the acoustic field. It appears that for both precision and efficiency reasons it is better to solve the plate equation exactly in time. Thus, the plate equation is resolved by modal reconstruction. The modes are first determined by the means of the finite element method. The crucial point of this particular time discretization, which couples two methods which are radically different, is to ensure the stability of the obtained scheme. As announced, this is obtained by an energy technique which is based on the variational formulation of the model.

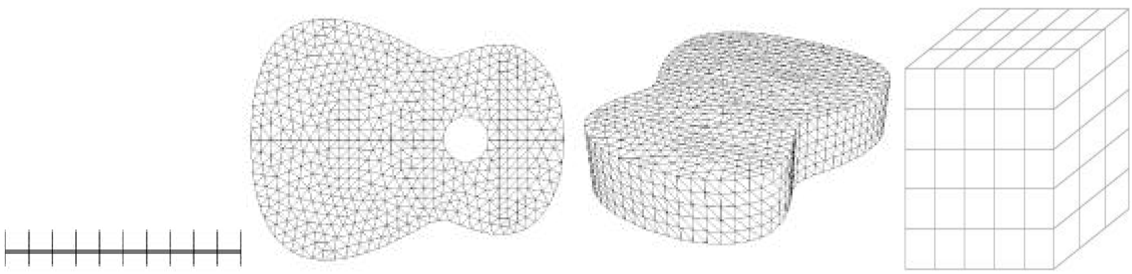


Figure 3: Meshes of string, top-plate, pressure jump and acoustic field.

RESULTS

For all numerical experiments, typical parameters measured on standard classical guitars were used. The purpose is not to reproduce one particular guitar but rather to evaluate the performance of model and method. Figure 4 shows two examples of admittance at a particular selected point on the bridge. Comparisons are made between top-plate only and the complete guitar with open strings. These results compare well with experimental results on real instruments. In our model, it is possible to let the plate vibrate *in vacuo* which, in turn, yields the comparison between material and radiation damping. The influence of position and properties of bracing and bridge on the produced sound was also investigated. It has been observed that the ear is very sensitive to the fine tuning of string and plate damping, which confirms the judgment of players. By varying the geometry of soundhole and cavity, it has been possible to investigate the influence of air modes on the temporal evolution of the sound. In particular, it has been observed that the tone envelope is significantly affected if some top-plate modes are strongly coupled to air modes. Figure 5 shows the evolution of directivity (rms sound pressure in dB) with time for a pluck on the open E (or 6^{th}) string. The time interval between consecutive pictures (from left to right and from top to bottom) is equal to 1.5 ms. In contrast to usual directivity patterns, which are drawn for a stationary field, these pictures give new insights with regard to the radiation properties of the instrument. Finally, Figure 6 yields the sound pressure field (linear scale in Pa) for a pluck on the open E (or 1^{st}) string. The time interval between consecutive pictures is 0.72 ms. Virtual recordings were made of the sound pressure at various positions around the guitar which yield different tone qualities.

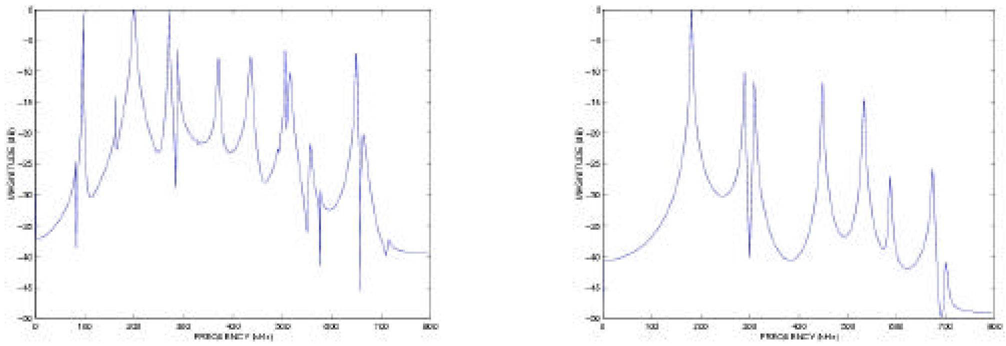


Figure 4. Admittance curves. Left: complete guitar. Right: top plate only.

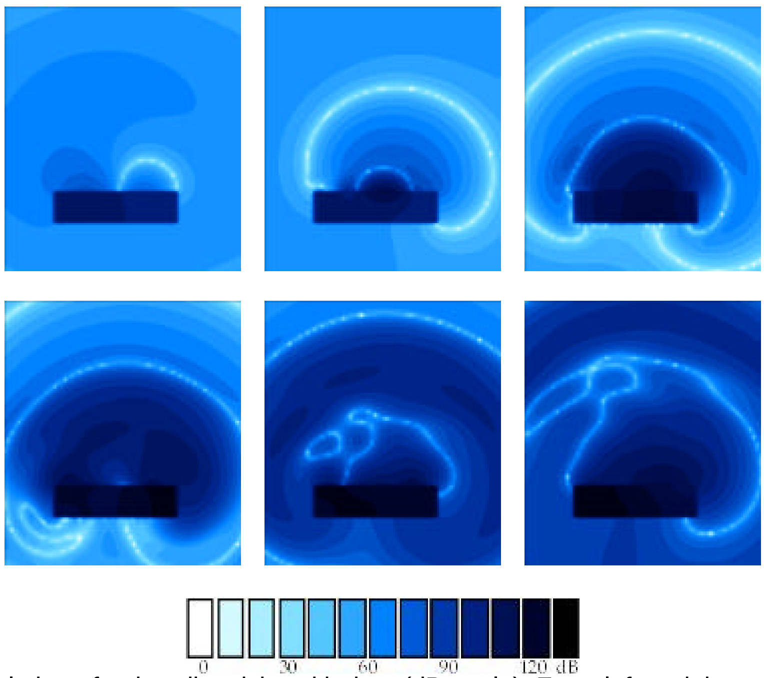


Figure 5. Evolution of guitar directivity with time (dB-scale). From left to right and from top to bottom.

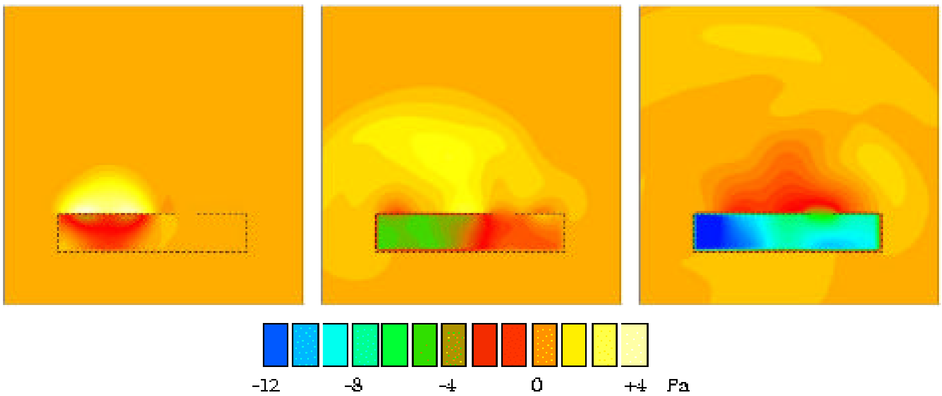


Figure 6. Evolution of sound pressure with time. From left to right. Linear scale (in Pa).

CONCLUSION

The representation of the orthotropic damped top-plate with bracing, coupled to cavity and strings allows a fair reproduction of eigenfrequencies, decay times and magnitudes of the partials observed in real guitar tones. The other results take benefit from the computation of the time history of the total acoustic free field inside and outside the instrument. The prime interest is, naturally, to synthesize realistic guitar sounds. Our technique allows to compute the evolution in time of quantities such as sound intensity, sound power, energy transfer between cavity and free field, directivity index and radiation efficiency. It is expected that the averaging of such quantities not only in frequencies, but also in time and spatial domain will be of particular interest with regard to makers, players and listener. The main novel point of the method is that complex geometries of a fluid-structure interaction problem can be tackled, by means of the fictitious domain method. In addition, the use of well-posed problems, based on energy estimation, guarantees the stability of the numerical schemes. The limitations of the methods are the following: first, the constraints due to the spatial mesh are such that the possibilities for modifying the bracing of the top plate are limited. Secondly, the string is assumed to vibrate in a plane, which makes it impossible to account for the double decay observed in real guitar sounds. Finally, air damping should be added in order to better account for the Q-factor of the Helmholtz resonance, and the string-finger interaction could be substantially improved. Other modifications, such as taking the motion of the back plate into account, does not bear any substantial difficulty.

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