A Combined Laminate and Honeycomb Wood Model for Softwood used for Numerical Optimization of a Violin Top

PACS no. 43.75.De, 62.20.Dc

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ABSTRACT A material model for softwood is presented where a combined lamiate and honeycomb model of wood is used. The model considers the influence from density variations in the annual rings, the cellular structure of wood, and the reinforcement from the wood rays. A numerical example that uses the material model is presented where stochastic optimization with the simulated annealing method (SA) is performed on a violin top made of Norway spruce *(Picea Abies).* The example shows that it is possible, through changes in thickness variables, to compensate for changes in vibration properties caused by a variation in the material parameters in the top.

INTRODUCTION

Wood material has a natural distribution of mechanical properties. Blanks for violin tops may have different density and elastic moduli, which in its turn means that the thickness or the rise of the arch must be adapted for each individual case. The vibration properties of the violin plates have been extensively studied both experimentally, e.g. Jansson *et al.* 1970 and numerically, e.g. Molin *et al.* 1984; Molin *et al* 1986 and Bretos Linaza *et al.* 1999. In the present work, optimization is used to give some suggestions on how to adapt the shape of the top for materials with different mechanical properties. The carefully chosen blanks make it meaningful to make more detailed material models than usual. In the material model presented in this paper, influence of density and volumetric variations in the annual rings, as well as the cellular structure of wood and the reinforcement from the wood rays are considered (influence from other cell types is not considered. A more thorough description of the wood structure is e.g. given in Kollman and Coté 1984).

WOOD MATERIAL MODEL

The special outtake of quarter sawn lumber as blanks for violin tops (Fig. 1) makes it possible to build up an adapted material model for the calculations. Furthermore, since the blank is rather thin and straight-grained, common laminate theory combined with the honeycomb model of the wood structure, gives a possibility to establish connections between the density of the material and the elastic properties.

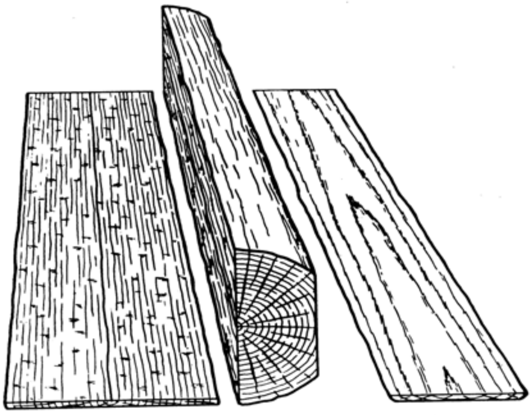

Fig. 1. A quarter sawn board to the left, showing the edge of the annual rings on its broad face, and a flat sawn board to right showing the side of the rings (adapted from Simpson 1991).

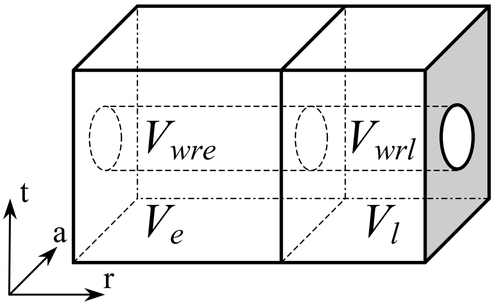

Fig. 2. Schematic model of the wood material with concentrated volume fractions of earlywood (to the left), latewood (to the right), and wood rays (cylinder in the middle).

List Of Symbols

E_a, E_r, E_t, elastic moduli in the axial (longitudinal), radial and tangential direction according to Figure 2.

E_{ae}, E_{al}, E_{re}, E_{rl} etc, elastic moduli for earlywood and latewood in the axial direction etc.

E_{wr} elastic modulus in radial direction for wood rays.

E_a^C, E_{tr}^C, elastic moduli of the cell wall in the axial and transverse directions.

E_{wr}^C, elastic modulus of the wood ray cell wall in radial direction.

G_{ar}, shear modulus in the *a-r* direction.

G_{are}, G_{arl}, shear moduli for earlywood and latewood.

G_{ar}^C, shear modulus of cell wall in the *a-r* direction.

V_e, V_l, V_{wr}, volume fractions of earlywood, latewood and wood rays in the blank, see Fig 2.

ρ, density of the blank (top).

ρ_e, ρ_l, ρ_{wr}, density of earlywood, latewood, and wood rays in the blank.

ρ_c, density of the cell wall.

Laminate Model Of Annual Rings

The orientation of the lighter and weaker earlywood, together with the darker and stronger latewood, builds up a laminate in a violin top according to Figure 3. Also, the thin material (about 3 mm) in the completed top makes the curvature of the annual rings negligible. The orientation of the fibres in the axial directions, and the lamina structure of earlywood and latewood in the transverse directions, would normally lead to an expected relationship of the elastic moduli as $E_a >> E_t > E_r$, but since the wood rays serve as a reinforcement in the radial direction, the relationship is $E_a >> E_r > E_t$. (Other factors are also believed to contribute to the relationship, such as different properties of the cell wall in the radial direction compared to the tangential, different angle of microfibrils in radial and tangential surfaces etc.) If the influence of the wood rays is neglected in the longitudinal and tangential directions, the elastic moduli in these directions, E_a and E_t, are given by “the rule of mixtures” (see e.g. Hull 1981) as

$$E_a = \frac{V_e}{V_e + V_l} E_{ae} + \frac{V_l}{V_e + V_l} E_{al} \tag{1}$$

and

$$E_t = \frac{V_e}{V_e + V_l} E_{te} + \frac{V_l}{V_e + V_l} E_{tl} \tag{2}$$

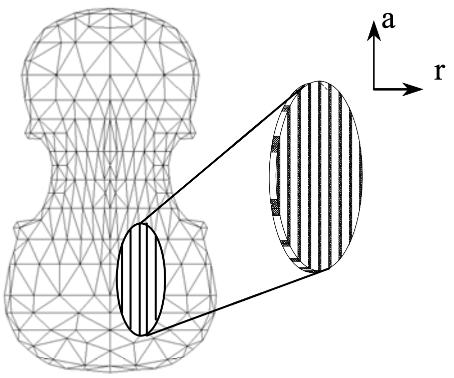

Fig. 3. Violin top with orientation of the annual rings with earlywood and latewood lamina. Tangential direction is perpendicular to the surface of the top.

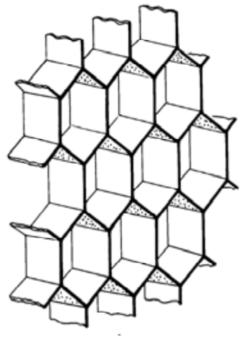

Fig. 4. Honeycomb model of the wood cells with cut in the transversal (radial and tangential) plane. Axial direction is perpendicular to these directions (adapted from Gibson and Ashby 1998).

(Volume fractions according to Fig. 2, more details is given in Carlsson and Tinnsten 2001) To estimate the influence of the wood rays in the radial direction, separate calculations must be made in the earlywood and the latewood. If elastic moduli for the wood ray reinforced material are notated by E'_{re} and E'_{rl}, and parallel displacements are assumed, common laminate theory gives

$$E'_{re} = V_{wre}E_{wr} + (1 - V_{wre})E_{re} \tag{3}$$

$$E'_{rl} = V_{wrl}E_{wr} + (1 - V_{wrl})E_{rl} \tag{4}$$

where the volume fractions of wood rays in earlywood and latewood, V_{wre} and V_{wrl}, are given by $V_{wri} = V_{wr}\dfrac{V_i}{V_e + V_l}, \quad i = e, l$ (i.e. earlywood and latewood, see Fig. 2). The total elastic modulus in the radial direction, E_r, is therefore, with (3) and (4), given by

$$\frac{1}{E_r} = \frac{V_e}{(V_e + V_l)}\frac{1}{E'_{re}} + \frac{V_l}{(V_e + V_l)}\frac{1}{E'_{rl}} \tag{5}$$

The shear modulus in the *a-r* direction, G_{ar}, is given by

$$\frac{1}{G_{ar}} = \frac{V_e}{(V_e + V_l)}\frac{1}{G_{are}} + \frac{V_l}{(V_e + V_l)}\frac{1}{G_{arl}} \tag{6}$$

Honeycomb Model Of Wood Cells

The cellular structure of the wood material has a great influence of the mechanical properties in different directions. On a millimetre scale, however, the honeycomb model gives a rather good description of the wood cells, see Figure 4. Furthermore, it gives a connection between the relative density of the wood and elastic moduli in different directions. Relative density is the density of the wood divided with the density of the cell wall, where the latter density is almost constant for different species of trees (a more detailed description of the mechanics of the honeycomb structure is given in Gibson and Ashby 1998).

From the mechanics of the regular honeycomb, the following relations can be found between the relative density of the wood and the elastic parameters.

$$E_{ai} = E_a^C \frac{\rho_i}{\rho_c}, \quad i = e, l \tag{7}$$

$$E_{ti} = 1.5 \cdot E_{tr}^{C} \left(\frac{\rho_i}{\rho_c} \right)^3, \quad i = e, l \tag{8}$$

$$E_{ri} = 1.5 \cdot E_{tr}^{C} \left(\frac{\rho_i}{\rho_c} \right)^3, \quad i = e, l \tag{9}$$

and for the elastic modulus of the wood rays in radial direction

$$E_{wr} = E_{wr}^{C} \frac{\rho_{wr}}{\rho_c} \cdot \tag{10}$$

For the shear modulus, G_{ar}, the relation is

$$G_{ari} = 0.5 \cdot G_{ar}^{C} \frac{\rho_i}{\rho_c}, \quad i = e, l \tag{11}$$

which completes the relations of interest in this work.

Combined Laminate And Honeycomb Model

If the equations from the previous sections of the laminate model and the honeycomb model are combined, a material model that considers both the annual rings and the cellular structure of wood can be achieved. Furthermore, the model also offers relations between the relative density of the structure of the top and its elastic moduli. For the axial, elastic modulus, E_a, equations (1) and (7) gives

$$E_a = \frac{E_a^C}{V_e + V_l} \left[V_e \frac{\rho_e}{\rho_c} + V_l \frac{\rho_l}{\rho_c} \right] \tag{12}$$

and it is interesting to note that for a given density ρ of the blank, the value of E_a, under some assumptions, is independent of the densities of earlywood and latewood, ρ_e and ρ_l, and their volume fractions V_e and V_l (this is only guilty for E_a). In tangential direction (2) and (8) gives

$$E_t = \frac{1.5 E_{tr}^{C}}{V_e + V_l} \left[V_e \left(\frac{\rho_e}{\rho_c} \right)^3 + V_l \left(\frac{\rho_l}{\rho_c} \right)^3 \right] \tag{13}$$

and similar relations can be derived for E_r and G_{ar}.

OPTIMIZATION AND ANALYSIS

As optimization routine, the simulated annealing method (SA) is used. The method usually requires a large number of function evaluations to find the optimum design, but it will usually find the global optimum even for problems with several local minima (more information of the SA method is given in Goffe *et al.* 1994 and Tinnsten and Carlsson 2001). This quality makes SA very suitable for acoustic optimization where numerous local minima occur. The initial (reference) top is characterized by a given rise of the arch of the top, an initial thickness distribution, and an initial set of material parameters. In the optimization, the first three eigenmodes are studied, (Fig. 5). For the purpose of optimization the initial material parameter set is replaced with a new set. The objective problem is to recover the first three eigenfrequencies in the top with the new parameter set by altering the thickness distribution. The shell thickness (t_i) at nodes along the bold line in Figure 5 are collected in one variable, and the thickness at nodes outside this line in another variable. The shell thickness at all other nodes are separately variables which gives a total number of variables I = 68. The optimization problem is formulated as:

Minimize

$$z = \sum_{k=1}^{3} \alpha_k \cdot \left(f_k^{initial} - f_k^{actual} \right)^2 \tag{14}$$

subjected to

$$0.9 \cdot t_i^{initial} \le t_i^{actual} \le 1.1 \cdot t_i^{initial}; \quad i = 1, I \tag{15}$$

where $f_k^{initial}$ is the eigenfrequency for the initial top, f_k^{actual} is the eigenfrequency for the top with the new material set and actual variable set (k=1,3), and I is the number of thickness variables. The variables α_k are so-called penalty variables.

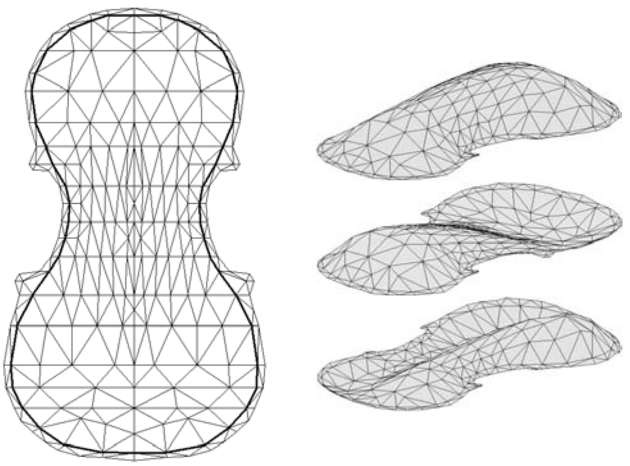

Fig. 5. Discretization of the violin top, and studied eigenmodes, where top is mode 1, middle is mode 2, and bottom is mode 3.

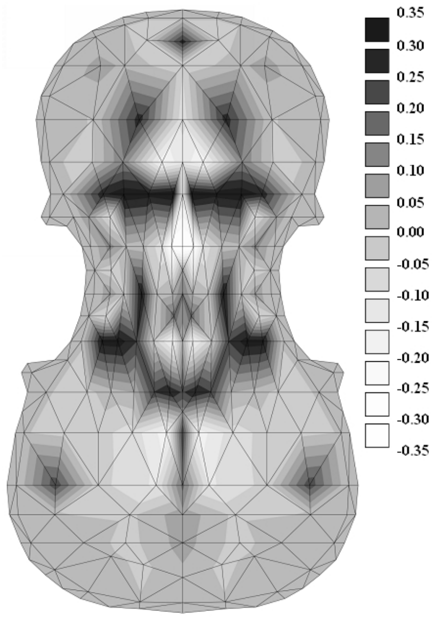

Fig. 6. Change in thickness distribution to reach optimum (values are in [mm] and should be added to the initial to give the optimum thickness distribution).

The violin top is discretizised according to Figure 5. In the FE-analyses involved in the optimization, a modified version of the finite element code FEMP (Nilsson and Oldenburg 1983), an orthotropic shell element is used (Molin *et al.* 1984; Tinnsten *et al.* 1999). The nodes along the bolded line are prevented to move perpendicular to the plane but free in all other directions.

RESULTS

Numerical calculations and optimization are performed on two tops with the same density but with one top (the reference top) with wide annual rings and another top with narrow annual rings. With material data according to Bucur (1995) and Gibson and Ashby (1998), the reference top with wide annual rings (see Carlsson and Tinnsten 2001 for more information) was given the following data: E_a = 9,470 MPa, E_r = 637 MPa, and G_{ar} = 644 MPa. Poissons ratio is given the (constant) value ν_{ar} = 0.03. With an initial thickness distribution, and the material parameters given above, the studied eigenfrequencies were determined to $f_1 = f_1^{initial} = 282.67\,\mathrm{Hz}$, $f_2 = f_2^{initial} = 508.81\,\mathrm{Hz}$ and $f_3 = f_3^{initial} = 544.37\,\mathrm{Hz}$. The other top with narrow annual rings got the following values of the elastic moduli: E_a = 9,470 MPa, E_r = 580 MPa, and G_{ar} = 666 MPa. Poissons ratio is the same as before, i.e. ν_{ar} = 0.03. The new material set together with the initial thickness resulted in the following eigenfrequencies: $f_1 = 280.47\,\mathrm{Hz}$, $f_2 = 505.54\,\mathrm{Hz}$ and $f_3 = 537.38\,\mathrm{Hz}$. These three frequencies are the first f_k^{actual}, $(k = 1{,}3)$, in the optimization process. After optimization of the top with narrow annual rings, a change in thickness distribution according to Figure 6 was proposed by the SA-routine. With the proposed thickness change of the top, the first three eigenfrequencies have changed to: $f_1 = f_1^{optimal} = 282.61\,\mathrm{Hz}$, $f_2 = f_2^{optimal} = 508.57\,\mathrm{Hz}$ and $f_3 = f_3^{optimal} = 544.28\,\mathrm{Hz}$ which gives a maximal difference of –0.048 % from the initial eigenfrequencies of the reference top.

DISCUSSION

The optimization problem was to minimize the difference in eigenfrequences for two violin tops with different material. Three eigenmodes were studied, and the problem was formulated as: minimize the quadratic sum of the difference of each eigenfrequency multiplied by a penalty parameter, by modifying the thickness of the top. The differences in eigenfrequency are within 0.05%, a result which promises much for a future extend of the model. As input values to the eigenfrequency analysis, a material model was developed which considers density variations in the annual rings, as well as the cellular structure of wood and the reinforcement in radial direction from the wood rays. It can be noted that the two tops, although they have common density and elastic modulus E_a, according to the material model have different elastic moduli in other directions which in its turn give other vibration properties. The boundary condition used in the analyses in this paper was primarily chosen to show the benefits of using optimization technique in eigenfrequency problems.

FUTURE CONSIDERATIONS

Much work remains to be done in order to understand the behaviour of violins and to be able to build good violins from blanks with different material properties. A whole violin must be studied, and it is also necessary to improve the material model to take care of wood structure in a more proper way.

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