DIRECT AND INVERSE SCATTERING OF TRANSIENT ACOUSTIC WAVES BY A SLAB OF RIGID POROUS MEDIA.

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ABSTRACT

This paper provides an experimental validation of the Pride-Lafarge [6,7] model of the acoustic wave propagation in porous material having a rigid frame. An experimental validation have been done for three plastics foams having different flow resistivity. A comparison between experimental and simulated attenuation data is given.

I. INTRODUCTION

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot [1,2] theory. In air-saturated porous media the structure is generally motionless and the waves propagates only in the fluid. This case is described by the model of an equivalent fluid in which the interactions between the fluid and the structure are taken into account in two frequency dependent response factors: the dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson [3] and the dynamic compressibility of the air included in the porous material $\beta(\omega)$ given by Allard [4].

Let us consider a homogeneous isotropic porous material with porosity ϕ saturated with a compressible and viscous fluid of density ρ_f and viscosity η. It is assumed that the frame of this porous solid is not deformable when it is subjected to an acoustic wave. It is the case for example for a porous medium which has a large skeleton density or very large elastic modulus or weak fluid-structure couplings. To apply the results of linear elasticity it is required that the wavelength of sound waves should be much larger than the sizes of pores or grains in the medium. In these porous materials acoustic waves propagates only in the fluid. They can be seen as an equivalent fluid, the density and the bulk modulus of which are "renormalized" by the fluid–structure interactions.

The basic equations of this model are the Euler equation and the law of the mass conservation associated with the behavior equation:

$$\boldsymbol{r}_f \boldsymbol{a}(\boldsymbol{w}) \frac{\partial \, \mathrm{v}_i(x,t)}{\partial \, t} = -\nabla_i \mathrm{p(x,\ t)}$$
$$\frac{\boldsymbol{b}(\boldsymbol{w})}{K_a} \frac{\partial \, \mathrm{p(x,\ t)}}{\partial \, \mathrm{t}} = -\nabla.\mathrm{v(x,\ t)}. \qquad (1)$$

In these relations, v and p are the particle velocity and the acoustic pressure, $\boldsymbol{r}_f$ and $K_a = gP_0$ are respectively the density and the compressibility modulus of the fluid, $\boldsymbol{a}(\boldsymbol{w})$ and $\boldsymbol{b}(\boldsymbol{w})$ are the dynamic tortuosity of the medium and the dynamic compressibility of the air included in the porous material. These two response factors are complex functions which heavily depend on the frequency $f = \boldsymbol{w}/2\boldsymbol{p}$

II. MODELS

A model initially developed by Johnson *et al* [3], and completed by Allard *et al* [4] and Lafarge *et al* [5] by adding the description of thermal effects gives the theoretical expressions of the dynamic tortuosity $\boldsymbol{a}(\boldsymbol{w})$ and the dynamic compressibility $\boldsymbol{b}(\boldsymbol{w})$:

$$\boldsymbol{a}(\boldsymbol{w}) = \boldsymbol{a}_\infty \left(1 + \frac{1}{jx} \sqrt{1 + j\frac{M}{2} x} \right),$$
$$\boldsymbol{b}(\boldsymbol{w}) = g - (g-1) / \left(1 + \frac{1}{jx'} \sqrt{1 + \frac{M'}{2} jx'} \right) \qquad (2)$$

Where $j^2=-1$, $x = \frac{\boldsymbol{w}\boldsymbol{a}_\infty \boldsymbol{r}_f}{\boldsymbol{s}\boldsymbol{f}}$, $M = \frac{8k_0 \boldsymbol{a}_\infty}{\boldsymbol{f}\Lambda^2}$, $x' = \frac{\boldsymbol{w}\boldsymbol{r}_0 k'_0 P_r}{\boldsymbol{h}\boldsymbol{f}}$, g represents the adiabatic constant, P_r the Prandtl number, $\boldsymbol{a}_\infty$ the tortuosity, k_0 the static permeability, k'_0 the thermal permeability, Λ and Λ' the viscous and thermal characteristics lengths [3,4]. The functions $\boldsymbol{a}(\boldsymbol{w})$ and $\boldsymbol{b}(\boldsymbol{w})$ express the viscous and thermal exchanges between the air and the structure which are responsible of the sound damping in acoustic materials. These exchanges are due on the one hand to the fluid–structure relative motion and on the other hand to the air compressions-dilatations produced by the wave motion. The parts of the fluid affected by these exchanges can be estimated by the ratio of a microscopic characteristic length of the media, as for example the sizes of the pores, to the viscous and thermal skin depth thickness $\boldsymbol{d} = (2\boldsymbol{h}/\boldsymbol{w}\boldsymbol{r}_0)^{1/2}$ and $\boldsymbol{d}' = (2\boldsymbol{h}/\boldsymbol{w}\boldsymbol{r}_0 P_r)^{1/2}$. For the viscous effects this domain corresponds to the region of the fluid in which the velocity distribution is perturbed by the frictional forces at the interface between the viscous fluid and the motionless structure. For the thermal effects, it is the fluid volume affected by the heat exchanges between the two phases of the porous medium. In this model the sound propagation is completely determined by the six following parameters: $\boldsymbol{f}, \boldsymbol{a}_\infty, \boldsymbol{s} = \boldsymbol{h}/k_0, k'_0, \Lambda$ and Λ'. The range of frequencies such that viscous skin thickness $\boldsymbol{d} = (2\boldsymbol{h}/\boldsymbol{w}\boldsymbol{r}_0)$ is much larger than the radius of the pores r $\frac{\boldsymbol{d}}{r} >> 1$, is called the low frequency range. For these frequencies, the viscous forces are important everywhere in the fluid. At the same time, the compression-dilatation cycle in the porous material is slow enough to favor the thermal exchanges between fluid and structure. The temperature of the frame is then practically unchanged by the passage of the sound wave because of the high value of its specific heat: the frame acts as a thermostat, and in this case the isothermal compressibility is directly applicable.

The low frequency approximation of the response factors $a(w)$ and $b(w)$ are given by tacking the limit $w \to 0$ in Eq. (2) which leads to the following expressions:

$$a(w) \approx \frac{hf}{jwr_f k_0} \qquad (3)$$
$$b(w) \approx g$$

In this domain of frequency the Euler equation is reduced to the Darcy's law which defines the static flow resistivity $s = \frac{h}{k_0}$.

When the frequency increases, the skin thickness becomes narrower and the viscous effects are concentrated in a small volume near the frame δ/r << 1. in this case the viscous effects in the fluid can be neglected: the fluid behaves almost like a perfect fluid (without viscosity). In this domain of frequencies the compression/dilatation cycle is much faster than the heat transfer between the air and the structure and in this case, it is a good approximation to consider that the compression is adiabatic.

The high frequency approximation of the responses factors $a(w)$ and $b(w)$ when $w \to \infty$ are given by the relations:

$$a(w) \approx a_\infty \left(1 + \frac{2}{\Lambda}\left(\frac{h}{jwr_f}\right)^{1/2}\right), \qquad (4)$$
$$b(w) \approx 1 + \frac{2(g-1)}{\Lambda'}\left(\frac{h}{jwP_r r_f}\right)^{1/2}.$$

Pride *et al* [6] and Lafarge *et al* [7] Model

Later on Pride *et al* [6] and Lafarge *et al* [7] give a correction of the previous model by adding the real part a_0 of the dynamic tortuosity at the low frequency limit, when: $w \to 0$, $a(w) \approx \frac{hf}{jwr_f k_0} + a_0$, and by adding two others parameters p and p' in the development at high frequency limit for the dynamic tortuosity and the dynamic compressibility, when $w \to \infty$, the expression of $a(w)$ and $b(w)$ becomes

$$\boldsymbol{a}(\boldsymbol{w}) \approx \boldsymbol{a}_\infty \left(1 + \frac{2}{\Lambda} \left(\frac{\boldsymbol{h}}{j\boldsymbol{w}\boldsymbol{r}_f} \right)^{1/2} + \frac{\boldsymbol{s}\boldsymbol{f}(1-p)}{j\boldsymbol{w}\boldsymbol{r}_0 \boldsymbol{a}_\infty} \right),$$

$$\boldsymbol{b}(\boldsymbol{w}) \approx 1 + \frac{2(g-1)}{\Lambda'} \left(\frac{\boldsymbol{h}}{j\boldsymbol{w}P_r \boldsymbol{r}_f} \right)^{1/2} + \frac{(g-1)\,\boldsymbol{h}\boldsymbol{f}\,(1-p')}{j\boldsymbol{w}\,k'_0\, P_r\, \boldsymbol{r}_0}. \quad (5)$$

with: $p = \dfrac{M}{4\left(\dfrac{\boldsymbol{a}_0}{\boldsymbol{a}_\infty} - 1\right)}$ and $p' = \dfrac{M'}{4(\boldsymbol{a}'_0 - 1)}$, $\boldsymbol{a}'_0$ is the thermal equivalent of $\boldsymbol{a}_0$.

The general expression of the dynamic tortuosity and the dynamic compressibility in this new model is given by:

$$\boldsymbol{a}(\boldsymbol{w}) = \boldsymbol{a}_\infty \left(1 + \frac{1}{jx} \left(1 - p + p\sqrt{1 + j\frac{M}{2p^2}x} \right) \right),$$

$$\boldsymbol{b}(\boldsymbol{w}) = g - (g-1) / \left(1 + \frac{1}{jx'} \left(1 - p' + p'\sqrt{1 + \frac{M'}{2p'^2} jx'} \right) \right). \quad (6)$$

This model verify the condition of causality if the singularities in the complex $\boldsymbol{w}$ plane are located in the lower half plane (Im($\boldsymbol{w}$) < 0) and verify the condition of long wavelength, which is specific to this problem if the singularities are in the imaginary axis [7], theses conditions restricts the values of p to :

$$p \geq \sqrt{\frac{M}{2}} - \frac{M}{4} \quad (7)$$

The expression of the attenuation $\boldsymbol{e}(\boldsymbol{w})$ which is the imaginary part of the wave number: $k(\boldsymbol{w}) = \dfrac{\boldsymbol{w}\boldsymbol{r}_f \boldsymbol{a}(\boldsymbol{w})\boldsymbol{b}(\boldsymbol{w})}{K_a}$ is given by

$$\boldsymbol{e}(\boldsymbol{w}) = -\frac{1}{2\,c_\infty} \left[\sqrt{\frac{2\,\boldsymbol{h}}{\boldsymbol{r}_f}} \left(\frac{1}{\Lambda} + \frac{g-1}{\sqrt{P_r}\Lambda'} \right) \sqrt{\boldsymbol{w}} + \frac{\boldsymbol{s}\boldsymbol{f}(1-p)}{\boldsymbol{r}_f \boldsymbol{a}_\infty} + \frac{\boldsymbol{h}\boldsymbol{f}(g-1)(1-p')}{k'_0\, P_r\, \boldsymbol{r}_0} + \frac{4(g-1)\boldsymbol{h}}{\Lambda\Lambda'\,\boldsymbol{r}_0 \sqrt{P_r}} \right] \quad (8)$$

In the case of Johnson [3] and Allard [4] model the attenuation is given by

$$\boldsymbol{e}(\boldsymbol{w}) = -\frac{1}{2\,c_\infty} \left[\sqrt{\frac{2\,\boldsymbol{h}}{\boldsymbol{r}_f}} \left(\frac{1}{\Lambda} + \frac{g-1}{\sqrt{P_r}\Lambda'} \right) \sqrt{\boldsymbol{w}} \right]. \quad (9)$$

3. ULTRASONIC MEASUREMENTS

As an application to this theory, some numerical simulations are compared to experimental results. Experiments are performed in air with two broadband Panametrics V386 piezoelectric transducers having a 150 kHz central frequency in air and a bandwith at 6 dB extending from 60 to

350 kHz. Pulses of 300 V are provided by a 5058PR Panametric pulser/receiver. Received signals are amplified up to 90 dB and filtered above 1MHz to avoid high frequency noise.

Fig.1, 2 and 3. shows a comparison between experimental and simulated data of the attenuation for different plastic foams M1, M2 and M3 (table. 1). The blue curve corresponds to the experimental data of the attenuation. The red dashed line curve corresponds to the simulated signal using Johnson-Allard model (Eq. 9) and the black dashed line curve corresponds to the simulated signal using Pride-Lafarge model (Eq. 8).

The values of p and p' used for the simulation are given in table. 1. These values are not well inverted because of the complexity of the inverse problem, the values of p' have been taken equal to those of p which is not verified in the general case. The sensitivity of p' is less important for the attenuation compared to the sensitivity of p, this is due to the viscous effects which are prevailing in the attenuation of the sound wave. The main difference between the black dashed line curve of the attenuation (Pride-Lafarge) model and the red dashed line curve (Johnson-Allard) model is situated in the gap at the vertical axis of the attenuation due to the added term of Pride-Lafarge model. This term is shown to be important to a best fit of the attenuation experimental curve.

Materials	α_∞	$\Lambda\ (\mu m)$	$\Lambda'\ (\mu m)$	$\sigma\ (Nm^{-4}s)$	ϕ	p	p'
M1	1.05	300	900	2500	0.98	0.8	0.8
M2	1.25	50	150	38000	0.92	1.1	1.1
M3	1.5	30	90	125000	0.82	1.3	1.3

Table.1. Parameters of the plastics foams.

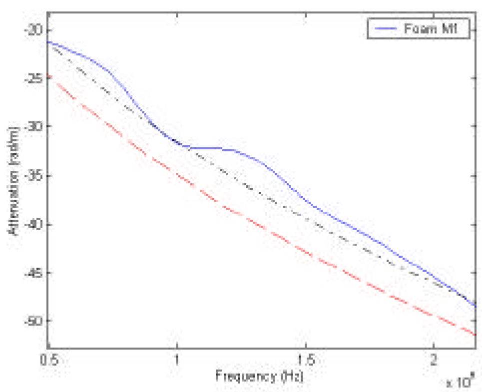

Fig. 1.

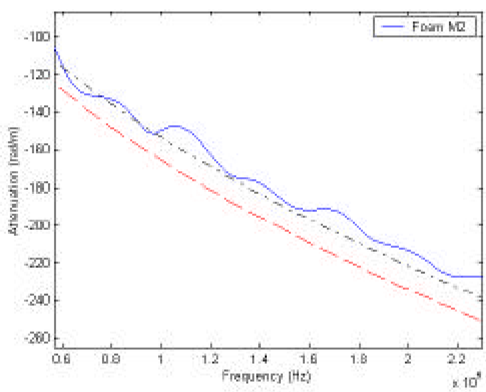

Fig. 2.

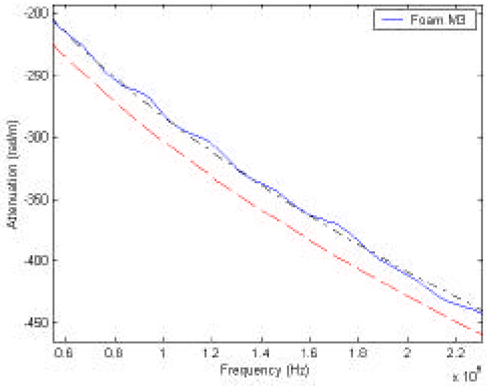

Fig. 3.

CONCLUSION

An experimental validation of the Pride-Lafarge model for the acoustic propagation in porous materials having a rigid frame is given. This model gives a better description of the propagation at high and low frequency range in the continuity of Johnson-Allard model. The values of the parameters p and p' used for the simulation are not well inverted. This is because of the complexity of the inverse problem due to the large number of the parameters. It is however shown the necessity of using the Pride-Lafarge model for a better description of the propagation.

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