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WAVE PROPAGATION IN A TIME-VARYING ELASTIC MEDIUM

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ABSTRACT.

Time-varying metamaterials are artificial media whose properties depend on the temporal variation of some of their physical parameters. Temporal discontinuities consist in an abrupt change of the constitutive parameters, which allow new forms of wave manipulation. Time-varying media are not conservative because the energy is supplied from an external source. An acoustic wave splits into two waves (a forward wave and a backward wave) when a temporal interface occurs throughout the spatial propagation domain. In this work, the propagation of elastic waves through a string with time-varying speed is studied in the context of elastic waves in a spatially homogeneous medium. Forward and backward wave dispersion coefficients are presented. The scattering coefficients of a single interface are extended to multiple interfaces using the transfer matrix method to analyze different speed profiles as a function of speed time. The theoretical results are validated with numerical FDTD numerical simulations.

RESUMEN. (Arial, línea 25, tamaño 10, alineado izquierda, interlineado sencillo, máximo 200 palabras).

Los metamateriales variables en el tiempo son medios artificiales cuyas propiedades dependen de la variación temporal de algunos de sus parámetros físicos. Las discontinuidades temporales consisten en un cambio abrupto de los parámetros constitutivos que permiten nuevas formas de manipulación de ondas. Los medios variables en el tiempo no son conservativos ya que la energía es introducida por una fuente externa. Una onda acústica se divide en dos (una onda hacia adelante y otra hacia atrás) cuando se genera una interfaz temporal en todo el dominio de propagación espacial. En este trabajo, se estudia la propagación de ondas elásticas a través de una cuerda con velocidad variable en el tiempo en el marco de ondas elásticas en un medio espacialmente homogéneo. Se presentan los coeficientes de dispersión de las ondas hacia adelante y hacia atrás. Los coeficientes de una única interfaz se extienden a múltiples interfaces por medio del método de matrices de transferencia para analizar diferentes perfiles de velocidad en función del de tiempo de velocidad. Los resultados teóricos se validan con simulaciones numéricas FDTD.

1. INTRODUCCIÓN

Recent advances in metamaterials have dramatically extended the range of modern metamaterial properties. Time-varying metamaterials are materials that show new and unusual properties due to the temporal variation of some of their physical parameters, induced by an external source of energy. In the newly developing topic of spacetime metamaterials, the study of wave phenomena with time-varying media has recently attracted interest [1,2]. The scientific and engineering communities have paid close attention to wave interactions caused by spacetime modulations in a number of domains, including elastic [7,8,9], acoustic [5,6], and electromagnetic [3,4] metamaterials.
Here, we introduce a rigorous and systematic framework for the study of a taut string with a multi-step time-varying speed.

Spacetime metamaterials unveil multiple physics and applications that can be exploited. Many interesting phenomena have emerged in time-varying systems like amplification of waves [10], sound circulation with non-reciprocal devices [11], asymmetric transmission [12], and non-Hermitian spacetime varying metamaterials [13]. The purely time-varying media, in which only time and not space changes with time, is one example of these spacetime metamaterials [14]. These systems are a pure spatial perturbation's dual. An imaginary boundary separates spatially two different media. Analogously, a temporal interface is defined as an abrupt change of the properties of the medium [15] at a specific instant time.
Interesting applications like improving the bandwidth of reflection of thin absorbers [16] and customizing the energy flow between two linked cavities [17] are made possible by changing the constitutive parameters of unbounded media. Purely temporal modulated media conserve momentum but not energy since they are spatially uniform. The energy of the field can increase leading to a new kind of amplification process [18]. Here, we present a theoretical framework for time-varying wave speed in an unbounded string. The scattering coefficients of a general speed profile described as a collection of temporal interfaces are provided through a matrix formalism.

2. THEORY

1.1. String model

We consider a string as an infinitely long, dispersionless, one-dimensional elastic system without losses. The wave equation is governed by the speed of the wave in the string, c(t). If the medium reaction is infinitely fast, a single temporal interface is defined as an abrupt change in the medium attributes occurring at time t = T. We consider a harmonic transverse wave propagating in the unbounded string frequency $\omega=2\pi/c_1$, where c_1 is the speed of the string. Since the wave propagates before the temporal interface, it is referred to as the *earlier wave* here. It is represented schematically in a spacetime diagram in Figure 1a) with a green arrow. Because the diagram is not in reciprocal space, the arrow does not represent the wavevector or any other vectorial magnitude of the wave. It is a visual representation of the wavefront's movement through time and space. A discontinuity in the properties of the string is produced by a sudden switch of the wave speed in the string from c_1 to c_2. The later-forward wave, y_f, and the later-backward wave, y_b, are two new waves that are produced as a result of the speed shift and propagate in opposing directions. Both waves are faster than the earlier wave because it is a slow-fast temporal interaction.

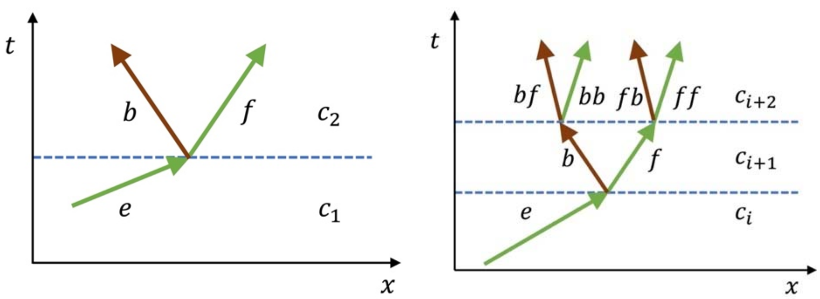

Figure 1 – Space-time diagram of the wave splitting produce by a) a single temporal interface and b) a temporal slab.

1.2. Time-Varying Transfer Matrix Method

A generalized temporal multilayer string's scattering behavior is assessed here using the Time-Varying Transfer Matrix Method [19,20]. At time Ti, the ith temporal interface is generated, and every position in the string switches from c_i to c_{i+1}. The two parameters that define this suddenchange in the properties of the medium are the speed contrast ($\gamma_i = c_{i+1} / c_i$) and the non-dimensional time ($\tau_i = \omega_i \cdot T_i$), where ω_i is the angular frequency of the wave at the ith medium. The wave generation process is illustrated schematically in the spacetime diagram at Fig. 1b), for N = 2, where two successive temporal interfaces are represented with two parallel discontinuous lines.

Like for a single interface, the backward and forward waves, denoted by y_b and y_f, split at each speed change and produce a forward and a backward wave, respectively, resulting in the generation of four new wave contributions, denoted by y_{ff}, y_{bf}, and y_{fb}. Observe that at the time frame with speed c_{i+1}, y_{bb} and y_{ff} propagate in the same way (shown in green), but y_{bf} and y_{fb} propagate in the other direction (shown in brown). The momentum conservation imposes continuity conditions to the transverse amplitude and stress waves before and after t = T_i at the *i*th temporal interface, assuming the string is spatially homogeneous.

The forward scattering coefficients at the ith interface can be obtained as:

$$F_i = \frac{y_{ff}^i + y_{bb}^i}{y_f^i + y_b^i} \qquad (1)$$

and backward coefficient as

$$B_i = \frac{y_{fb}^i + y_{bf}^i}{y_f^i + y_b^i} \qquad (2)$$

The set of forward and backward scattering coefficients before (F_i and B_i) and after (F_{i+1} and B_{i+1}) the ith temporal interface can be arranged in matrix form in order to relate a through the 2 x 2 transfer matrix $M_{i,i+1}$:

$$\begin{bmatrix} F_{i+1} \\ B_{i+1} \end{bmatrix} = M_{i,i+1} \begin{bmatrix} F_i \\ B_i \end{bmatrix} \qquad (3)$$

where

$$M_{i,i+1} \begin{bmatrix} f_{\gamma,\tau} & b_{\gamma,\tau}^* \\ b_{\gamma,\tau} & f_{\gamma,\tau}^* \end{bmatrix} = \begin{bmatrix} F_i \\ B_i \end{bmatrix}, \qquad (4)$$

the forward scattering coefficient is

$$f_{\gamma,\tau} = \frac{y_f}{y_e} = \frac{\gamma+1}{2\gamma} e^{-j\tau(1-\gamma)}\ , \qquad (5)$$

and the is the backward scattering coefficient is

$$b_{\gamma,\tau} = \frac{y_b}{y_e} = \frac{\gamma-1}{2\gamma} e^{-j\tau(1+\gamma)}\ . \qquad (6)$$

If γ = 1, there is no medium discontinuity, and the scattering coefficients correspond as expected to a time-invariant and homogeneous string with a freely propagating wave: $f_{1,0}$ = 1 and $b_{1,0}$ = 0, and there is no frequency conversion $\omega_f = \omega_b = \omega_e$.

2. TEMPORAL STRING

1.2. Single interface

The wave amplitude scattering and the energy coefficients at the temporal interface at T = 0 are shown in Fig. 2, as a function of the speed contrast g. The results are compared to numerical simulations based on Finite-Difference Time-Domain (FDTD) technique. In the case of $\gamma = 1$, the medium remains unchanged, and the wave propagation is homogeneous both in spatial and temporal dimensions. Obviously and as expected, the backward coefficient is null, $|b_{1,\tau}| = 0$, because no speed contrast exists and the forward wave has the same amplitude as the earlier wave, $|f_{1,\tau}|$. Due to the discretization of the numerical scheme, the temporal interface in FDTD simulations is not perfectly abrupt as the time step is not null. However, as it is significantly small compared to the typical temporal magnitudes (the period of the wave $2\pi/\omega_e$ and the temporal slab duration T) in the model, it can be neglected. The string with length L = 500m is long enough to avoid the interference of waves reflected at the boundaries. The time step is set to $\Delta t = 5 \cdot 10^{-6}$s. The sudden change of medium properties may lead to a strong instability of the simulations. Therefore, to ensure convergence, the domain has been discretized on time with enough steps to represent the highest frequency in the domain. The spatial step in the mesh is assured to satisfy, at least, the Courant stability criterion, $\Delta x = 1$ cm. Numerical and analytical results match both the amplitude and energy coefficients. Regardless of the speed contrast γ, the forward energy flux is always $\Pi_f \geq 1$. Thus, a change of speed with time implies a net energy flux of the medium provided to the forward wave.

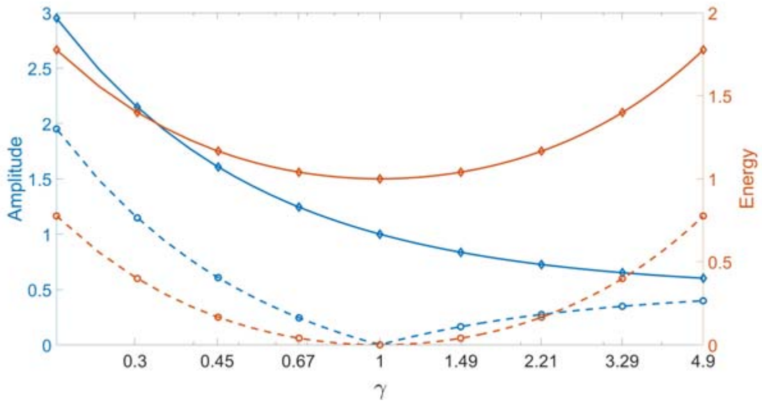

Figure 2 – Wave amplitude and energy scattering coefficients of a single interface with speed contrast γ

1.2. Temporal slab

We consider a string that experiences a sudden change in speed and then eventually returns to its initial state. The temporal slab is a time-varying structure made up of two reciprocal, non-simultaneous temporal interfaces with contrast speeds γ and γ^{-1}. Here, we examine how an earlier wave propagates over a slab of duration T that is made up of the same first and final media and is distinguished by a speed contrast (γ) and a time delay (τ). Two matrices are used to apply the TV-TMM to the time slab, one at time t = 0 s and the other at time t = T, with contrast speeds of γ and γ^{-1}, respectively.

3. FDTD SIMULATIONS

Wave propagation in a homogeneous medium with a time slab has been studied numerically using the Finite-Difference Time-Domain (FDTD) method. The interface in the numerical simulation is not completely abrupt since the time step is not zero. However, because it is so much smaller than the two common temporal scales (represented by the wave period and time slab duration), it can be disregarded. The string is 500 meters long, or L. The mesh's spatial step is set to x = 1 cm. The simulations' numerical stability may become unstable due to a sudden change in the medium's parameters. As a result, the domain has been discretized in time accurately enough to represent the domain's maximum frequency in order to assure convergency. The spatial step in the mesh is assured to satisfy, at least, the Courant stability criterion, $\Delta t = 5*10^{-6}$s (Courant number set to 0.5).

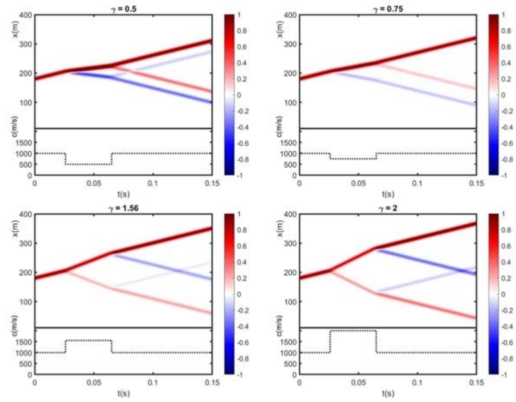

Figure 4 – FDTD simulations of a Gaussian pulse in different temporal slabs.

First, a wave train propagating in a homogeneous medium with a time slab is analyzed. The excitation signal is a step-windowed continuous wave with a frequency of f = 10 Hz. The amplitude and the temporal part contribution of wave phase are represented in the spacetime diagram for a mountain slab, i.e., a time slab with a slow-fast-slow profile, $\gamma = 2$, in Fig. 3a) and 3c) and a valley slab with a fast-slow-fast profile, $\gamma = 0.5$, in Fig. 3b) and 3d). The amplitude of the wave train moving along a string with contrast speed $\gamma = 2$ in the positive direction of x is seen in Fig. 3a). The wave is first created at time zero and starts to move freely at a constant speed of 1000 m/s.

After some time, the speed is abruptly changed to c_2 = 2000 m/s, two scattered waves are generated, and the backward wave interferes in space and time with the forward wave. The spatial and temporal extension of the interference fringes depends on the duration of both the train and the temporal slab and the speed contrast. The change of speed due to the temporal interface can be observed as a variation of the slope of the boundaries of later train waves with respect to the earlier train wave. After, a new switch of speed is produced in the second temporal interface of the temporal slab. The speed changes again to c_1 = 1000 m/s and a new split of waves is produced leading to two backward waves, y_{fb} and y_{bf}, and two forward waves, y_{bb} and y_{ff}. In Fig. 3b) the same wave train propagates with the same initial speed c_1 = 1000 m/s, but this time in a valley slab with contrast speed $\gamma = 0.5$. Due to the slowness of the waves inside the slab, the interference pattern is more important than in the mountain slab. The phase of the wave is represented with a cyclic color scale between $-\pi$ and π. For simplicity, the spatial variation of the phase has been eliminated and only the variation of phase due to the advance of time is considered. The temporal evolution of color with propagation is opposite for the forward waves (green-blue-red) than for the backward waves (red-blue-green) meaning an increase and decrease of phase with time, respectively. In the interference region, the phase of the resultant pattern is governed by the wave with a higher amplitude. Thus, the phase at the interference after the first temporal interface is increasing (as the forward wave), and after the second interface is decreasing for the backward wave (as the backward-forward) and is increasing for the forward wave (as the forward-forward wave).

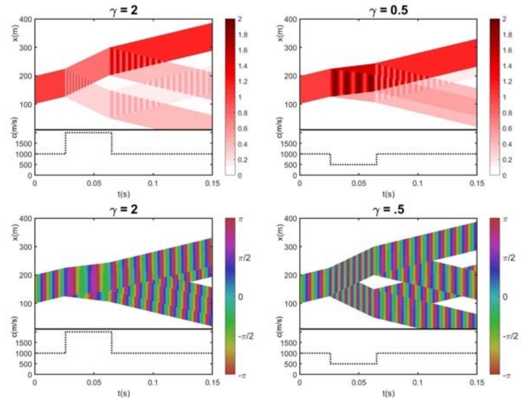

Figure 3 – FDTD simulations of a wave train in two different temporal slabs.

The propagation of a Gaussian wave with unitary amplitude through a valley slab is simulated and represented in the space-time diagram in Fig. 4 for different temporal slabs with contrast speed γ . Note that the color of the signal is red for the positive phase and blue for the negative phase and it is darker for higher wave amplitudes. As the duration of the pulse is much smaller than the temporal slab, the superposition of waves is very short in time and space and the interference fringes are not present. In this case, it is more relevant to analyze the four waves generated by the temporal slab separately. The color of the signal represents the amplitude of the pulse. It is red for the positive phase and blue for the negative phase and it is darker for higher values of the amplitude module. Phase shifts are produced only for backward waves at fast-slow temporal interfaces, $\gamma < 1$. Forward pulsating waves do not change the phase in a temporal slab.

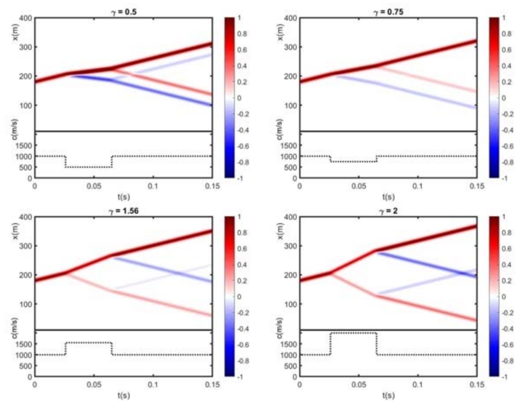

Figure 4 – FDTD simulations of a Gaussian pulse in different temporal slabs.

3. CONCLUSIONS

A matrix formalism is proposed to account for the scattering coefficients of a homogeneous, dispersionless string without losses with a generic profile of temporal interfaces. The constitutive parameter is the speed of transverse waves. The temporal slab is analyzed for increasing and decreasing contrast speed. Numerical simulations for wave trains present interference patterns in space and time between forward and backward waves. The space-time diagram shows the effect in amplitude and phase produced by different temporal slabs in short pulses is analyzed in the space-time diagram.

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REFERENCES

[1] C. Caloz and Z.-L. Deck-Léger, Spacetime Metamaterials-Part I: General Concepts, *IEEE Trans. Antennas Propag.* 68, 1569–1582, 2020.

[2] S. Yin, E. Galiffi and A. Alù, Floquet metamaterials, *eLight* 2:8, 2022.

[3] N. Engheta, Metamaterials with high degrees of freedom: Space, time, and more, *Nanophotonics* 10(1), 639–642, 2021.

[4] E. Galiffi et al., Photonics of Time-Varying Media, *Advanced Photonics* 4, 1, 014002, 2022.

[5] A. Hayrapetyan, K. Grigoryan, R. Petrosyan, and S. Fritzsche, Propagation of sound waves through a spatially homogeneous but smoothly time-dependent medium, *Ann. Phys.* 333, 47–65, 2013.

[6] N. Kruss and J. Paulose, Nondispersive One-Way Signal Amplification in Sonic Metamaterials, *Physical Review Applied* 17, 024020 (2022)

[7] G. Trainiti and M. Ruzzene, Non-reciprocal elastic wave propagation in spatiotemporal periodic structures, *New Journal of Physics*. 18 083047 (2016).

[8] D. Torrent, W. J. Parnell, and A. -N. Norris, Loss compensation in time-dependent elastic metamaterials, *Physical Review B* 97, 014105, 2018.

[9] M. Farhat et al., Spacetime modulation in floating thin elastic plates, *Physical Review B* 104, 014308, 2021.

[10] J. B. Pendry, E. Galiffi and P. -A. Huidobro. Gain in time-dependent media - a new mechanism, *Journal of the Optical Society of America B* 38, 11, 2021.

[11] R. Fleury, D. -L. Sounas, and A. Alù. Subwavelength ultrasonic circulator based on spatiotemporal modulation. *Physical Review B* 91, 174306, 2015.

[12] Q. Wang et al., Acoustic asymmetric transmission based on time-dependent dynamical scattering, *Scientific Reports* 5, 10880, 2015.

[13] X. Wen, Unidirectional amplification with acoustic non-Hermitian space−time varying metamaterial, *Communications physics* 5:18, 2022.

[14] J. T. Mendoça and A. -M. Martins. Temporal beam splitter and temporal interference, *Physical Review A* 68, 043801, 2003.

[15] V. Pacheco-Peña and N. Engheta, Antireflection temporal coatings, *Optica* 7, 323, 2020.

[16] H. Li and A. Alù, Temporal switching to extend the bandwidth of thin absorbers, *Optica* 8, 1, 2021.

[17] Y. Mazor, M. Cotrufo and A. Alù, Unitary Excitation Transfer between Coupled Cavities Using Temporal Switching, *Physical Review Letters* 127, 013902, 2021.

[18] T. -T. Koutserimpas, A. Alù, and R. Fleury, Nonreciprocal Gain in Non-Hermitian Time-Floquet Systems, *Phys. Rev. A* 97, 1, 013839, 2018.

[19] D. Ramaccia, A. Alù, A. Toscano, Temporal multilayer structures for designing higher-order transfer functions using time-varying metamaterials. *Appl. Phys. Lett.* 118, 101901, 2021.

[20] D. -W. Wright and R. S. C. Cobbold, Acoustic wave transmission in time-varying phononic crystals, *Smart Mater. Struct*. 18, 015008, 2009.