

ESTIMATION OF THE STRUCTURAL DYNAMIC RESPONSE USING THE TRANSMISSIBILITY CONCEPT

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Abstract

The dynamic behaviour of a structure can be modelled in various ways. In practice, the response model is often used, through the measurement of frequency response functions (FRFs), as a theoretical or even an analytical model may be very difficult or impossible to establish, reproducing adequately the experimental results. It happens, however, that once in service it may be impossible to take measurements on the structure, as the co-ordinates may no longer be accessible. The concept of transmissibility applied to multiple degree of freedom systems may help in the estimation of the desired FRFs, due to some particular transmissibility properties, a subject that has been published recently by the authors. In this paper the underlying transmissibility theory and the relevant properties are summarised and some experimental examples are given to illustrate the usefulness of the technique.

Keywords: Transmissibility, Structural Modification.

1 Introduction

The transmissibility for a single degree-of-freedom system, when its base is moving harmonically, is defined as the ratio between the modulus of the response amplitude and the modulus of the imposed motion amplitude.

It is possible to extend the transmissibility to a system with N degrees-of-freedom, relating a set of unknown responses to another set of known responses, for a given set of applied forces. The papers by Ewins and Liu [1] and Varoto and McConnell [2] extend the initial concept to N degrees-of-freedom systems in a limited way, the former using a definition that makes the calculations dependent on the path taken between the considered co-ordinates involved, the latter by making the restriction that the set of co-ordinates where the displacements are known coincide with the set of applied forces.

The transmissibility concept seems of great interest in cases when in field service one cannot measure the responses at some co-ordinates of the structure. If the transmissibility can be evaluated beforehand

in the laboratory or theoretically (numerically), then by measuring in service some responses one would be able to estimate the responses at the inaccessible co-ordinates.

2 Fundamental formulation

Ribeiro [3] proposed a general answer to the problem, based on harmonically applied forces (easy to generalize to periodic ones). Let \mathbf{F}_A be a vector of magnitudes of the applied forces at co-ordinates A , \mathbf{X}_U a vector of unknown response amplitudes at co-ordinates U and \mathbf{X}_K a vector of known response amplitudes at co-ordinates K ; then,

$$\mathbf{X}_U = \mathbf{H}_{UA} \mathbf{F}_A \quad (1)$$

$$\mathbf{X}_K = \mathbf{H}_{KA} \mathbf{F}_A, \quad (2)$$

where \mathbf{H}_{UA} and \mathbf{H}_{KA} are the receptance frequency response matrices relating co-ordinates U and A , and K and A , respectively. Eliminating \mathbf{F}_A between (1) and (2), it follows that

$$\mathbf{X}_U = \mathbf{H}_{UA} \mathbf{H}_{KA}^+ \mathbf{X}_K \quad (3)$$

or

$$\mathbf{X}_U = \mathbf{T}_{UK}^{(A)} \mathbf{X}_K \quad (4)$$

where \mathbf{H}_{KA}^+ is the pseudo-inverse of \mathbf{H}_{KA} . Thus, the transmissibility matrix is defined as:

$$\mathbf{T}_{UK}^{(A)} = \mathbf{H}_{UA} \mathbf{H}_{KA}^+ \quad (5)$$

The set of co-ordinates (A) where the forces can be applied need not coincide with the set of known responses (K). The only restriction is that the number of K co-ordinates must be greater or equal than the number of A co-ordinates, to allow the inversion of \mathbf{H}_{KA} .

Although the transmissibility curves look very similar to frequency-response-functions, their properties are very different. For instance the peaks in the transmissibility functions have nothing to do with the resonances of the system and the anti-peaks have nothing to do with the anti-resonances. However, comparing the various transmissibility functions, one can observe that the peaks always occur at the same frequencies.

A study on the properties of transmissibility functions can be found in [4]. One of the most interesting ones is the fact that the transmissibilities do not depend on the magnitude of the applied forces, they only depend on their positions (co-ordinates A). This is easy to understand, as the transmissibilities are obtained eliminating the force vector between equations (1) and (2).

There are many cases where the harmonic input functions are not used, even in controlled laboratory conditions. The transmissibility concept can be extended to non-harmonic excitations, using the spectral densities of the response measurements. The transmissibility matrix established from the spectral density functions is identical to the one from *FRFs* or Fourier spectra responses only [4].

2.1 Identified properties

In Ref. [4] three properties have been identified, from which the most important here are:

Property 1 - The values of the transmissibility matrix do not change if some modification is made on the mass values of the system where the dynamic loads can be applied – subset A.

Property 2 - The values of the transmissibility matrix do not change if some modification is made on the stiffness of springs connecting co-ordinates of subset A.

Note that these properties result from the fact that to add masses and/or stiffnesses at co-ordinates A is equivalent to apply forces, and the transmissibilities are invariant with respect to the amplitude of those forces.

2.2 Calculating the FRFs of the Modified System using the Transmissibility Matrix

From eq. (5), one has $T_{UK}^{(A)} = \mathbf{H}_{UA} \mathbf{H}_{KA}^+$. It has just been stated that if modifications are carried out at co-ordinates A, the transmissibility matrix remains constant. Thus, for the modified system, one has:

$$\mathbf{T}_{UK}^{(A)} = \mathbf{H}'_{UA} \mathbf{H}'_{KA} \quad (6)$$

Therefore,

$$\mathbf{T}_{UK}^{(A)} = \mathbf{H}_{UA} \mathbf{H}_{KA}^+ = \mathbf{H}'_{UA} \mathbf{H}'_{KA} \quad (7)$$

Thus, if the receptance matrix relating co-ordinates K and A of the modified system (\mathbf{H}'_{KA}) is known, the receptance matrix relating co-ordinates U and A (\mathbf{H}'_{UA}) can be estimated:

$$\mathbf{H}'_{UA} = \mathbf{T}_{UK}^{(A)} \mathbf{H}'_{KA} \quad (8)$$

In the next section an experimental example is presented.

3 Experimental case study

3.1 Original structure

The experimental case study consists of a beam with six equal masses attached (fig. 1). This structure is herein referred to as the *original structure*, and was built to allow simple mass and stiffness modifications at some co-ordinates/regions. The blocks are initially attached to the structure through a bolt, having a washer between each block and the beam. The washers act as local point springs. By removing the washers the two blocks are compressed directly to the beam surface, leading to an increase in the local stiffness. In fig.1 it is also shown the accelerometer locations and the points where the forces can be applied.

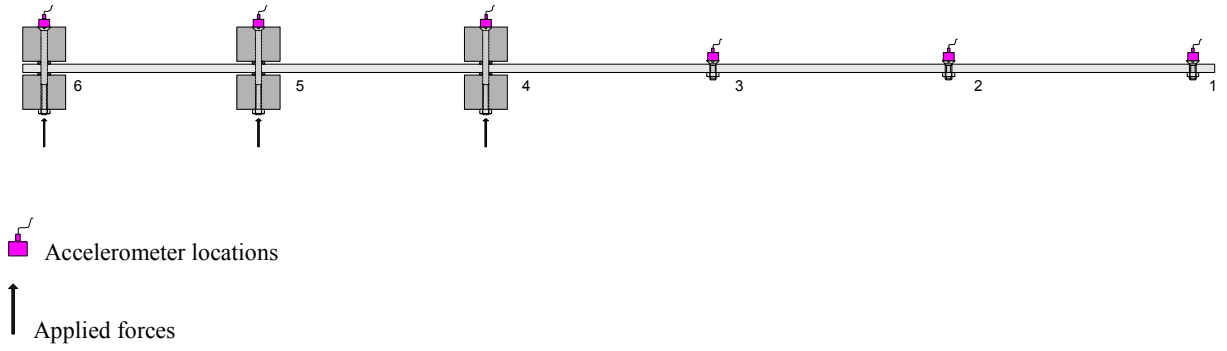


Figure 1 - Original structure.

The subsets of *known* and *unknown* responses are assumed as:

$$\{X_K\} = \begin{Bmatrix} \{X_2\} \\ \{X_4\} \\ \{X_6\} \end{Bmatrix}, \{X_U\} = \begin{Bmatrix} \{X_1\} \\ \{X_3\} \\ \{X_5\} \end{Bmatrix} \quad (9)$$

The vector $\{F_A\}$ contains the loads which can be applied (even if some of them are null in certain cases)

$$\{F_A\} = \begin{Bmatrix} \{F_4\} \\ \{F_5\} \\ \{F_6\} \end{Bmatrix} \quad (10)$$

According to eq. (5), and considering the above-defined subsets, the transmissibility matrix is given by:

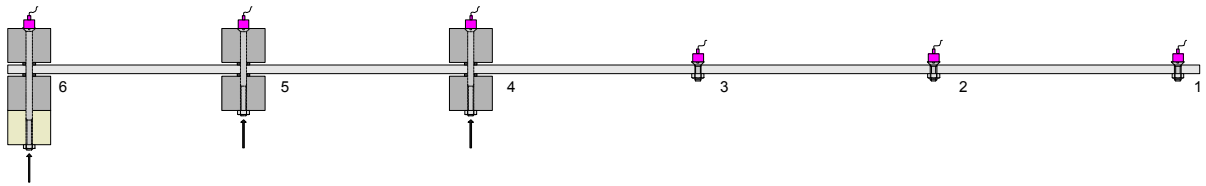
$$\begin{bmatrix} T_{12}^{(A)} & T_{14}^{(A)} & T_{16}^{(A)} \\ T_{32}^{(A)} & T_{34}^{(A)} & T_{36}^{(A)} \\ T_{52}^{(A)} & T_{54}^{(A)} & T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{34} & H_{35} & H_{36} \\ H_{54} & H_{55} & H_{56} \end{bmatrix} \begin{bmatrix} H_{24} & H_{25} & H_{26} \\ H_{44} & H_{45} & H_{46} \\ H_{64} & H_{65} & H_{66} \end{bmatrix}^+ \quad (11)$$

3.2 Modified structures

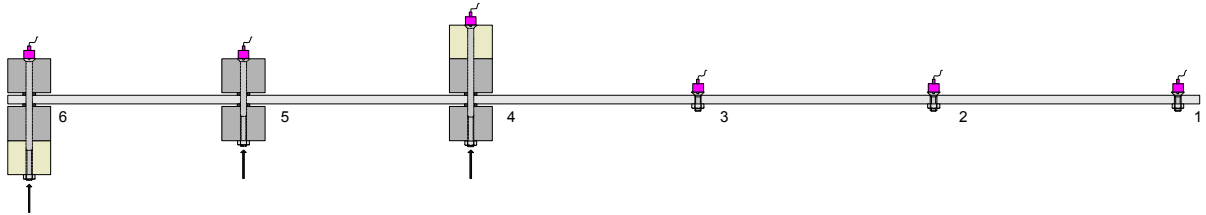
In order to prove that it is possible to obtain the FRFs in certain locations of modified systems, provided that the transmissibility matrix of the original system is known, some modifications have been made in the original structure. These modifications are made by adding masses or by changing the stiffness in certain regions.

The mass modifications made on the original structure are present in fig. 2 and the stiffness modifications in fig. 3.

Mass modified structure 1



Mass modified structure 2



Mass modified structure 3

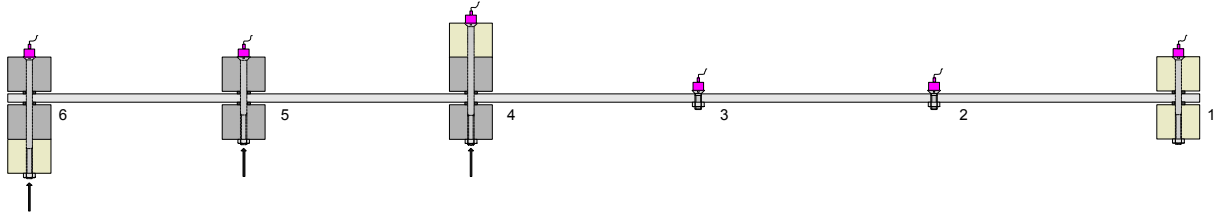
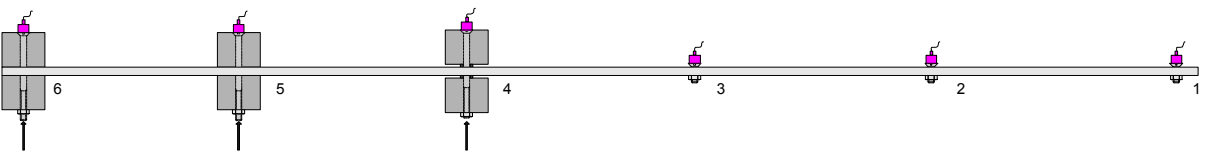


Figure 2 - Mass modified structures

Stiffness modified structure 4



Stiffness modified structure 5



Figure 3: Stiffness modified structures

The measured receptances H_{14} are presented in fig. 4 for the various modified structures. Fig. 4a) shows the influence of the mass modifications, i. e., the decreasing of the natural frequencies, whereas in fig. 4b) it can be observed the influence of the stiffness modifications resulting in higher natural frequencies.

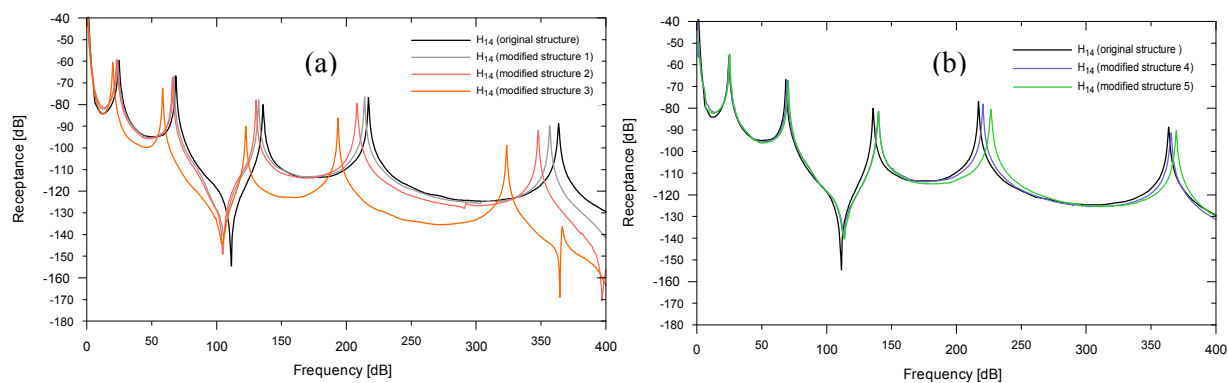


Figure 4: Measured receptances H_{14} for the original structure and modified structures: a) mass modification, b) stiffness modification.

In figs. 5 and 6 two transmissibility functions, T_{52} and T_{32} , respectively, are presented for the original and modified structures.

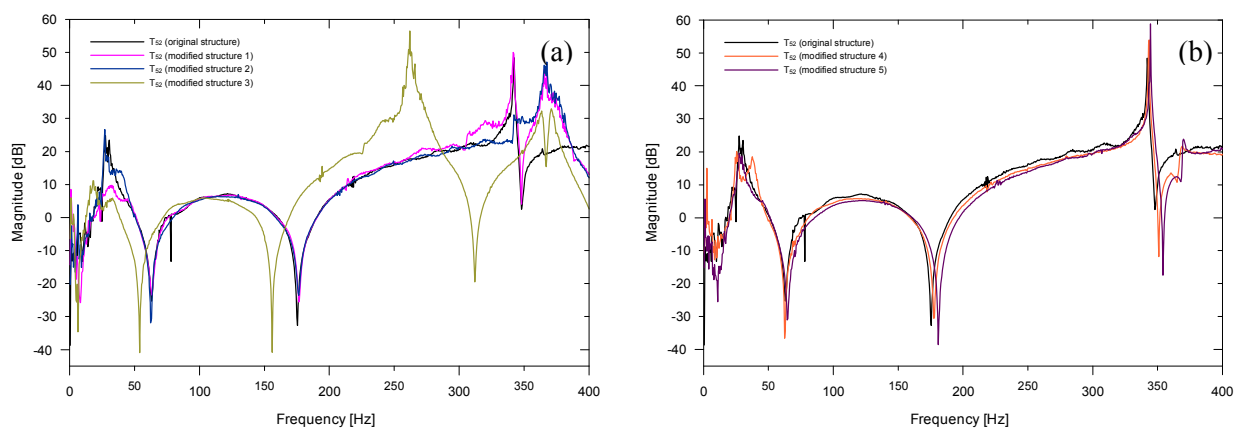


Figure 5: T_{52} Transmissibility obtained for original structure and a) mass, b) stiffness modified structures.

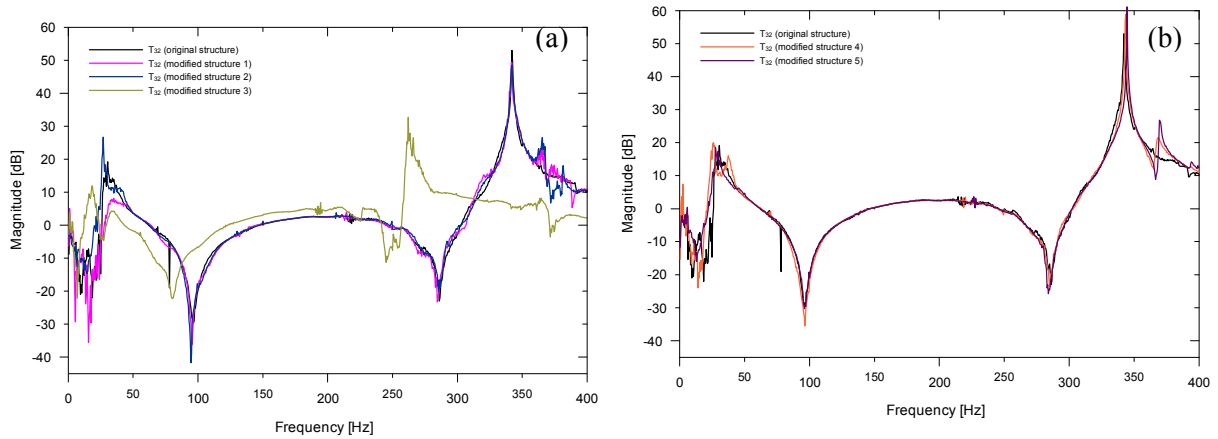


Figure 6: T_{32} Transmissibility obtained for original structure and a) mass, b) stiffness modified structures.

Remarks

It can be observed that in situations when the mass modifications on the original system are made on the values of the masses corresponding to the set of coordinates A (where the dynamic loads can be applied) the transmissibility values do not change, *modified structures 1 and 2*, figs. 5 and 6 (a). It can also be observed that when the modification on the original system is made on the stiffness region connecting co-ordinates of subset A the transmissibility values do not change, *modified structures 4 and 5*, figs.5 and 6 (b).

3.3 Estimation of FRFs using the Transmissibility concept

In figs. 7, 8, 9 and 10, three receptances H'_{14} , H'_{35} and H'_{56} , estimated for the modified structures 1, 2, 4 and 5, respectively, by using eq. (8), are presented and compared with the measured receptances on the actual modified structures.

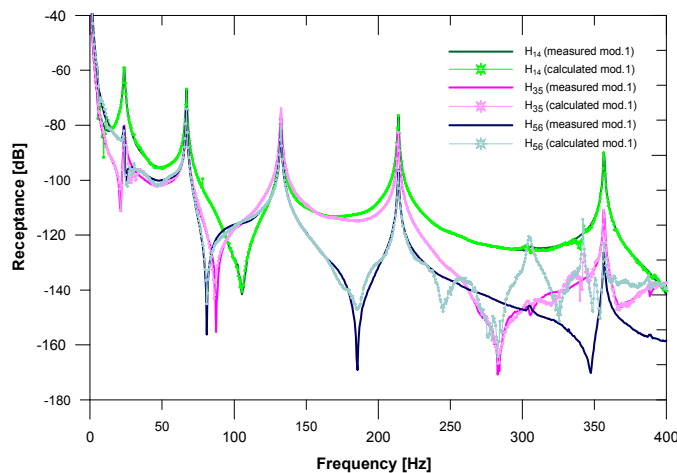


Figure 7: Measured and estimated receptances H_{14} , H_{35} and H_{56} for the mass modified structure 1.

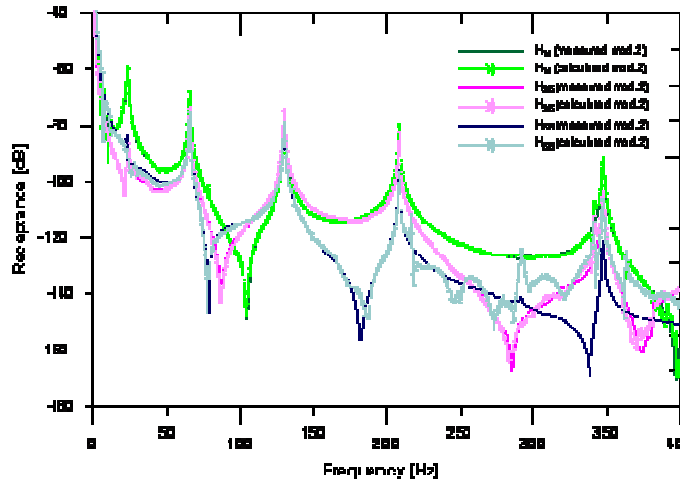


Figure 8: Measured and estimated receptances H_{14} , H_{35} and H_{56} for the mass modified structure 2.

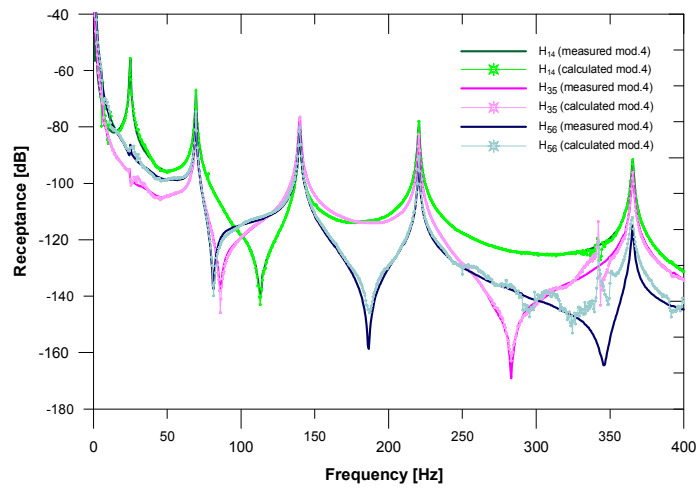


Figure 9: Measured and estimated receptances H_{14} , H_{35} and H_{56} for the stiffness modified structure 4.

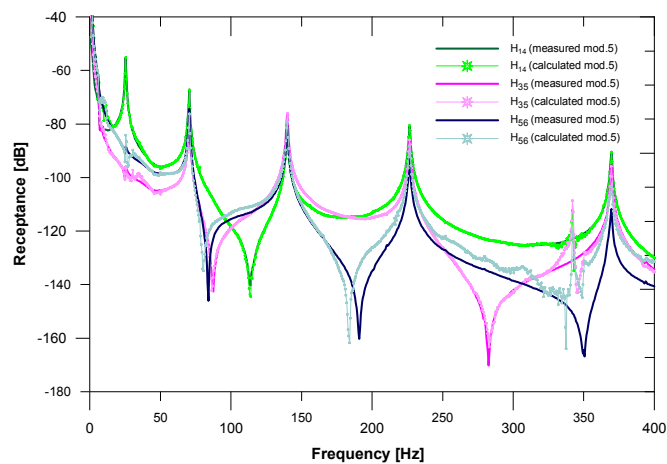


Figure 10: Measured and estimated receptances H_{14} , H_{35} and H_{56} for the stiffness modified structure 5.

Remarks

It can be observed that the estimated FRFs of the various modified structures compare well with the FRFs measured on the actual modified structures. This is not the case for Receptance H_{56} , mainly at the anti-resonances as the frequency increases.

4 Conclusions

In this work it has been shown that it is possible to estimate the FRFs related to some points of interest that are not physically accessible. By using the important properties associated with the transmissibility matrix, obtained by measuring the FRFs at certain points due to some applied forces on an actual original structure, together with a set of FRFs measured on any modified structure, it is possible to estimate response data at certain inaccessible locations of different mass and stiffness modified systems.

Acknowledgements

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