



NONLINEAR MIXING: EVALUATION OF NONLINEAR ULTRASONIC PARAMETER

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Abstract

An experimental validation of a technique called noncollinear mixing which studies ultrasonic waves nonlinearity is proposed at this work. The technique is based on ultrasonic wave generation from nonlinear interaction between two ultrasonic waves, which have different propagation frequencies values and it does not occur with an exclusive linear mechanics process. Murnaghan constitutive nonlinearity m parameter has been computed applying Korneev theoretical development. The parameter values have been obtained from the wave analysis of four different types of interactions, depending on the waves nature (P-P, P-S, S-P and S-S wave interactions). Fundamentals about wave interactions, waves mode conversion and ultrasonic waves propagation have been regarded for the specimen design. Finally Murnaghan m parameter values have been obtained for aluminum and compare with bibliography values.

Keywords: Nonlinear ultrasonics, Nonlinear mixing, Nonlinear acoustic parameters, Ultrasonic waves interaction.

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1 Introduction

Materials acoustic nonlinear properties have been widely studied by a long list of authors in mediums like fluids [1], rocks and concrete [2], metallic materials [3,4,5] or biologic materials [6]. The classical cited works are based on propagation and evaluation of ultrasonic singular waves, where the single variation suffered by the waves is due to test material nonlinear properties. In opposite to these classic nonlinear studies, Jones and Kobett [7] developed a new theory for the measurements of TOECs (Landau A, B and C parameters [8]), which was based on ultrasonic waves interaction. This new approach born from the fact that the material itself makes waves, which paths intersect, could interact between them [4]. Under certain circumstances this interaction could induce a third wave generation which propagation frequency and wave vector is the sum of interacting waves frequencies and waves vector.

Waves mixing techniques has two principal advantages over conventional ultrasonic lineal harmonics generation techniques. The first one consist on a minor sensibility to system nonlinearity due to: spatial selectivity (the nonlinear interaction is limited to the common beam path region), the modal selectivity (the mixing signal has a different mode that the incident waves), the frequency selectivity (the mixing signal frequency could be separate to the incident waves harmonics if incident waves frequency are

chosen in order to be different) and the directional selectivity (the mixing signal is propagated with a different path than the incident waves). At second place, in opposite to ultrasonic harmonic generation techniques, subjacent system nonlinearity level could be directly measured through the separately response of each incident wave without the supposed interaction.

At this work a study of Murnaghan m nonlinear acoustic parameter [9] for aluminum is proposed based on Korneev waves interaction model [10] through the immersion noncollinear waves mixing experimental technique, and results obtained are compared with previous m parameters published by D. Muir (2009) [11] and M. Stobbe (2005) [12].

2 Theoretical background

Murnaghan m parameter values have been experimentally obtained through the application of V. Korneev theoretic model [10] at this work. In opposite to the cited model, which proposes an analysis of the scattered waves generated by direct transmission waves interaction, the experimental tests carried out are based on an immersion ultrasonic transmission technique. Samples are submerged in an immersion tank at test, so to adapt Korneev model to proposed technique is necessary to apply the physics fundamentals shown in Figure 1, which are explained briefly at next subsections.

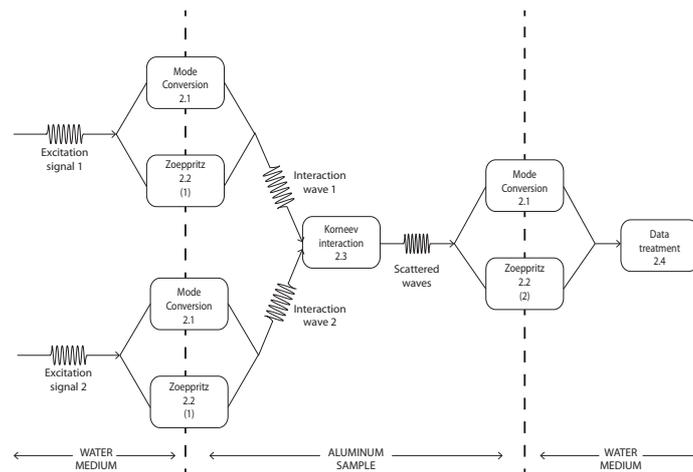


Figure 1 – Physics fundamentals applied to the signals in order to obtain Aluminum Murnaghan ‘ m ’ parameter.

2.1 Mode conversion

Excitation waves are propagated through water medium with θ_1 direction to aluminum samples surface normal. When these P-waves find in their path the sample surface, a wave mode conversion occurs and a P-wave and a S-wave are generated with θ_2 and φ_2 angles to the samples surface normal. To calculate that angles Snell Law shown at equation 1 is applied.

$$v_{p-water} \sin(\theta_2, \varphi_2) = (v_{p-al}, v_{s-al}) \sin(\theta_1). \quad (1)$$

Where v_i are the wave velocities for the different mediums, and θ_1 , θ_2 and φ_2 are the wave angles to the samples surface normal. Depend of the type of interaction studied, in order to eliminate undesired waves, is necessary to applied critical angle criteria, which is deduced from the equation 1 itself.

2.2 Transmission Zoeppritz coefficient

Waves amplitude is modified when mode conversion occurs too. To calculate waves amplitudes when they suffer a medium change it is necessary to apply transmission coefficients. These coefficients are obtained through the application of the Zoeppritz equations [13]. Waves amplitude must be multiplied by the coefficient Tp obtained as solution of Zoeppritz equations (for the test carried out the coefficient takes the value $Tp=0.16$) to the excitation signals amplitude when they change from water medium to aluminum medium (see (1) in Figure 1).

In order to obtain m parameter values through the application Korneev interaction model, it is necessary to substitute the amplitude of the waves generated at the interaction into the interaction volume (as it is a direct transmission interaction model). Thus, hydrophone acquired signals need to be corrected by the application of transmission coefficients in order to obtain waves amplitudes at the interaction volume. Because of transmission coefficients depend on wave nature, waves angle to the material surface and medium properties, each wave generated at the four types of interactions transmission coefficient take a singular value (see (2) in Figure 1). Applied transmission coefficient values for each scattered wave analyzed are collected in Table 1.

Table 1 – Transmission coefficient applied to each scattered wave analyzed.

Type of wave interaction	Scat. direction	Transmission coefficient
P - P	Ψ_1	0.03
P - P	Ψ_2	0.10
P - S	Ψ_3	0.06
P - S	Ψ_1	0.23
P - S	Ψ_4	0.29
S - P	Ψ_1	0.55
S - P	Ψ_3	0.28
S - P	Ψ_4	0.31
S - S	Ψ_1	0.16
S - S	Ψ_2	0.19
S - S	Ψ_4	0.23

2.3 Korneev interaction model

Under explicit circumstances two propagated waves, which propagation frequencies are respectively ω_1 and ω_2 , could interact and generate secondary waves (called scattered waves) with a propagation frequency ω_g . The angle (α) necessary to make interaction possible between *interaction wave 1* and *interaction wave 2* is obtained through equation 2 [10].

$$\alpha = \left(\frac{v_1 v_2}{2\omega_1 \omega_2} \right) \left(\left(\frac{\omega_g}{v_g} \right)^2 - \left(\frac{\omega_1}{v_1} \right)^2 - \left(\frac{\omega_2}{v_2} \right)^2 \right). \quad (2)$$

Where:

- ω_g , ω_1 and ω_2 are propagation frequencies of *scattered waves*, *interaction wave 1* and *interaction wave 2* respectively.



- v_g , v_1 , and v_2 are propagation velocities of *scattered waves*, *interaction wave 1* and *interaction wave 2* respectively.

Once waves interaction occurs four *scattered waves* are generated into the interaction volume. The angle between *scattered waves* and *interaction wave 1* (which is used as reference) is called scattered wave direction (ψ_i) and is obtained solving equation 3 [10]:

$$\psi = a \tan \left(\frac{\pm \frac{v_1}{v_2} d \sin \alpha}{1 \pm \frac{v_1}{v_2} d \cos \alpha} \right). \quad (3)$$

Where $d = \omega_1 / \omega_2$.

The equation 3 has four solutions corresponding with the directions of the four *scattered waves* generated at the interaction (ψ_1 , ψ_2 , ψ_3 and ψ_4 directions).

The nature of the *scattered waves* generated at the interaction depends on interaction waves nature. A S-waves are generated from a P-P waves interaction, P-waves are generated from a P-S waves interaction, P-wave are generated from a S-P waves interaction and P-wave are generated from a S-S waves interaction too.

2.4 Data treatment

Following Korneev interaction model, scattered waves amplitude (u) are obtained through the equation 4:

$$u(r, t) = W_g^\xi \frac{A_1 A_2}{r} V_g^\xi. \quad (4)$$

Where V_g^ξ is the interaction volume (which is approximated by a r radius sphere), r is the wave beam radius, u is the amplitude of the scattered waves generates into interaction volume, A_1 and A_2 are interaction waves amplitude and W_g^ξ is a coefficient which value for each interaction is shown at Table 2.

Table 2 – Parameters involved at equation 4.

Type of interaction	W_g^ξ	D_i	Other constants
P-wave vs P-wave	$W_g^\xi = (-D_s(1+d)\gamma^2 m 2 \sin(2\alpha_{p-p})) / 2$	$D_s = \frac{d}{4\pi(\lambda+2\mu)} \left(\frac{\omega_1}{v_s} \right)^3$	$\gamma = \frac{v_s}{v_p}$
P-wave vs S-wave	$W_g^\xi = -\frac{D_p \sin \alpha_{p-s}}{\gamma^3(1+d)} (d\gamma + q)m$	$D_s = \frac{d}{4\pi(\lambda+2\mu)} \left(\frac{\omega_1}{v_p} \right)^3$	$q_{p-s} = \cos \alpha (2d\gamma \cos \alpha + d^2 + 2\gamma^2)$
S-wave vs P-wave	$W_g^\xi = -\frac{D_p \sin \alpha_{s-p}}{\gamma^3(1+d)} (d\gamma + q)m$	$D_s = \frac{d}{4\pi(\lambda+2\mu)} \left(\frac{\omega_1}{v_p} \right)^3$	$q_{s-p} = \cos \alpha (2d\gamma \cos \alpha + 1 + 2\gamma^2 d^2)$
S-wave vs S-wave	$W_g^\xi = -\frac{D_p(1+d)}{2\gamma^2} m \cos(2\alpha)$	$D_s = \frac{d}{4\pi(\lambda+2\mu)} \left(\frac{\omega_1}{v_p} \right)^3$	$d = \frac{\omega_1}{\omega_2}$



Murnaghan m parameter is obtained substituting Table 2 parameters into equation 4. The parameter expression for each type of interaction is shown at equation 5 (for P-wave – P-wave interaction), equation 6 (for P-wave – S-wave interaction), equation 7 (for S-wave – P-wave interaction) and equation 8 (for S-wave – S-wave interaction).

$$m_{p-p} = -\frac{8\pi ur(\lambda + 2\mu)v_s^3}{A_1 A_2 V_g^\xi d\omega_1^3 (1+d)\gamma^2 \sin(2\alpha)} \quad (5)$$

$$m_{p-s} = -\frac{4u(1+d)rv_p^3\gamma^3(\lambda + 2\mu)\pi}{A_1 A_2 dV_g^\xi \omega_1^3 (d\gamma + \cos\alpha(d^2 + d\gamma^2 + 2d\gamma\cos\alpha))\sin\alpha} \quad (6)$$

$$m_{p-s} = -\frac{4u(1+d)rv_p^3\gamma^3(\lambda + 2\mu)\pi}{A_1 A_2 dV_g^\xi \omega_1^3 (d\gamma + \cos\alpha(1 + 2d^2\gamma^2 + 2d\gamma\cos\alpha))\sin\alpha} \quad (7)$$

$$m_{s-s} = -\frac{8\pi ur\gamma^2(\lambda + 2\mu)v_p^3}{A_1 A_2 V_g^\xi d\omega_1^3 (1+d)\gamma^2 \cos(2\alpha)} \quad (8)$$

The m values are obtained substituting u scattered waves amplitudes registered by the hydrophone at test after transmission coefficient application, being known rest of equations 5-8 parameters.

3 Materials and methods

In order to validate the new experimental method, aluminum has been chosen as test material due to large list of Landau [8] and Murnaghan [9] nonlinear parameters experimental studies.

At present work Murnaghan m parameter values have been obtained through four different ultrasonic waves interaction (P-wave and P-wave interaction, P-wave and S-wave interaction, S-wave and P-wave interaction and S-wave and S-wave interaction) using V. Korneev formulation [10]. Waves interaction occurs inside of the aluminum specimens. Because of the proposed experimental technique is based on immersion ultrasonic wave propagation technique, samples design is subjected to several physic fundamentals: wave mode conversion, Zoeppritz wave transmission equations, Korneev waves interaction model and again Zoeppritz wave transmission equations and wave mode conversion.

In order to design the specimens, as Figure 2 shows, first step is to compute ultrasonic wave angle (α) interaction through the Korneev [10] proposed model using equation 2 (1). Then, following Snell's law (shown at equation 1), it is necessary to obtain the relationship between ultrasonic waves transmitted through water medium and that wave transmitted through aluminum media affected by wave mode (2). Finally, samples design concludes substituting that angles expression into critic angle equation in order to eliminate undesired waves, which can interfere at waves interaction (3).

Experimental tests are carried out once samples design process is completed. Four scattered waves are generated from each waves interaction when the *interaction waves* interact between them. As Figure 2 shows the four scattered waves directions are obtained applying equation 3. Those waves are subjected to mode conversion process when they change from aluminum medium to water medium. Registering the scattered waves at water and applying data analysis explained at section 2.4, m Murnaghan parameter is obtained to each type of interaction.

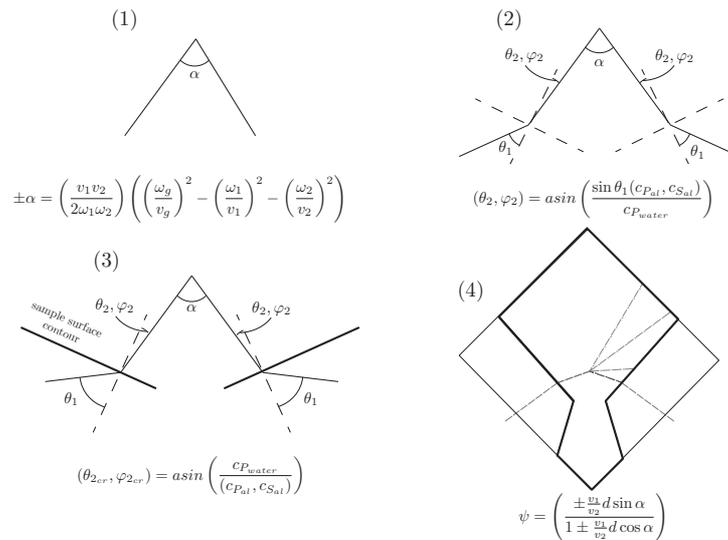


Figure 2 – Aluminum samples design process.

Waves parameters, which are shown at Table 3, are substituted at equation 2 in order to obtain theoretically relative *interaction waves* directions through excitation signals direction.

Table 3 – Interaction waves and scattered waves parameters.

In 1	In 2	Out	V ₁ (m/s)	V ₂ (m/s)	V _g (m/s)	ω _g (MHz)
P-wave	P-wave	S-waves	6250	6250	3100	2
P-wave	S-wave	P-waves	6250	3100	6250	10.5
S-wave	P-wave	P-waves	3100	6250	6250	10.5
S-wave	S-wave	P-waves	3100	3100	6250	10.5

Where *In 1* and *In 2* are the *interaction waves* with 4.25 MHz and 6.25 MHz excitation frequency respectively, *Out* is the *scattered waves* nature, V₁ and V₂ are the *interaction waves* propagation velocity, V_g is the *scattered waves* propagation frequency and ω_g is the *scattered waves* propagation frequency.

Interaction α angle is obtained replacing these values into equation 2. For the four types of wave interactions α values are: α_{p-p} = 140.28 degrees, α_{p-s} = 128.44 degrees, α_{s-p} = 91.19 degrees and α_{s-s} = 124.38 degrees.

Relationship between generated waves propagation direction (θ₁) and interaction waves propagation direction (θ₂ for P-wave interaction wave and φ₂ for S-wave interaction wave) are established following equation 1. θ₂ angles and φ₂ angles depending on type of interaction are collected in Table 4 which have been obtained replacing waves parameters at equation 1:

Table 4 – θ₂ and φ₂ values for each type of interaction.

Type of interaction	θ ₁ (wave 1) (deg)	θ ₁ (wave 2) (deg)	Scattered wave 1 direction (deg)	Scattered wave 2 direction (deg)
P (1) vs P (2)	5	5	θ ₂ = 21.59	θ ₂ = 21.59
P (1) vs S (2)	5	20	θ ₂ = 21.59	φ ₂ = 45.76
S (1) vs P (2)	20	5	φ ₂ = 45.76	θ ₂ = 21.59
S (1) vs S (2)	20	20	φ ₂ = 45.76	φ ₂ = 45.76

Once all restriction angles have been calculated, samples design is obtained applying critical angle criteria and reformulating the problem as a geometric problem. Samples design is proposed for each type of interaction as Figure 3 shows, assuming samples surface as Snell law model surface and satisfying that generated waves and interaction waves form angles collected at Table 4 with the samples surface normal and interaction waves form each α angle between them.

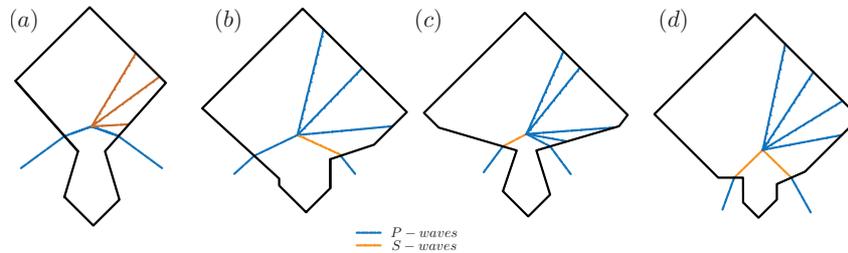


Figure 3 – Aluminum samples final design: P-wave – P-wave interaction (a), P-wave – S-wave interaction (b), S-wave – P-wave interaction (c) and S-wave – S-wave interaction (d).

Scattered waves directions for each type of interaction are computed using equation 3. Scattered waves directions values are shown at Table 5.

Table 5 – Scattered waves directions each type of wave interaction.

Type of interaction	ψ_1 (deg)	ψ_2 (deg)	ψ_3 (deg)	ψ_4 (deg)
P-wave and P-wave	39.14	16.51	-39.14	-16.51
P-wave and S-wave	50.48	20.41	-50.48	-20.41
S-wave and P-wave	33.68	12.72	-33.68	-12.72
S-wave and S-wave	23.56	38.21	-23.56	-38.21

Scattered waves are propagated through the sample and after suffering a mode conversion when they change from aluminum medium to water medium they are registered by the hydrophone. Murnaghan m parameter for each type of interaction is obtained applying data analysis shown at section 2.4.

4 Experimental tests

Tests were carried out using the scheme connection shown at Figure 4. As it could be seen at Figure 4, two wave generators *Agilent 33250A* were used in order to generate the two *excitation signals* with different propagation frequencies. Wave generator 1 was configured with 20 generated wave cycles, 4.25 MHz excitation frequency and burst mode on. Wave generator 2 was configured with 80 generated wave cycles, 6.25 MHz excitation frequency and burst mode on too.

Each wave generator was connected to one amplifier. The first one was the *Amplifier Research 25A250A* and the second one was the *Amplifier Research 150A100B*. Both of them were configured to provide +22 dB of gain to each wave.

The two amplifiers are connected to two actuators *Olympus panametrics-NDT, V310, 5 MHz / 0.25''*, 686665 which resonance frequency is located at 5 MHz. The two actuators were attached on an own-designed support. The support consists on two principal parts. The first one was build with two aluminum segments linked together through an axis which allow horizontal rotate between them, thus permitting to control the relative beams angle. On these metallic arms are fixed two PVC hollow

cylinders where actuators are placed. The second part of the device is an aluminum plate where samples were located.

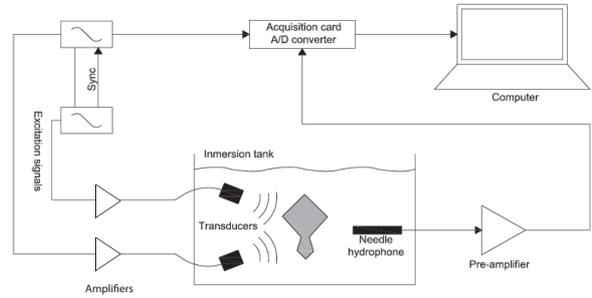


Figure 4 – Connection scheme for immersion noncollinear mixing.

Excitation signals were generated and after they were propagated following the path water-aluminum sample-water they were registered by the hydrophone (*Onda HNR-0500 4 Mhz – 10 Mhz*). Acquired signals were amplified by the preamplifier *Olympus ultrasonic preamplifier 5675, 172 42.5 dB*, which provide a gain of +42.5 dB to them.

Finally signals are received by the acquisition card and stored at the computer, where they were analyzed in order to obtaining the Murnaghan m parameter through immersion noncollinear technique.

5 Results

At this section aluminum Murnaghan m parameter values are obtained through the analysis of *scattered waves* acquired. *Scattered waves* frequencies are dependent of the interaction nature. Thus for each type of interaction *scattered waves* are $\omega_g = \omega_2 - \omega_1$ for P-wave – P-wave interaction and $\omega_g = \omega_2 + \omega_1$ for the rest of wave interactions .

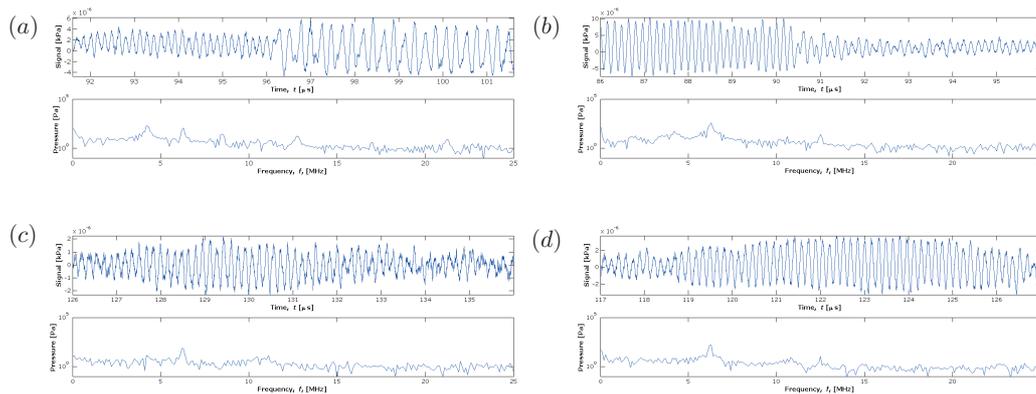


Figure 5 –Scattered waves shape at time domain and frequency domain: P-wave – P-wave interaction scattered wave (a), P-wave – S-wave scattered wave (b), S-wave – P-wave interaction scattered wave (c) and S-wave – S-wave interaction scattered wave (d).

Amplitudes appearing in equations 5, 6, 7 and 8 are obtained extracting waves amplitude at ω_g frequencies from FFT analysis of the *scattered waves* registered at each test. As example, Figure 5



shows some *scattered waves* shape from the four type of interaction in time domain and frequency domain.

Amplitudes substituted at equations 5,6, 7 and 8 are collected at Table 6. Substituting these values into cited equations aluminum Murnaghan m parameter is obtained. Aluminum m values for each type of interaction are collected in Table 6 too.

Table 6– *Scattered waves* amplitude registered by the hydrophone and aluminum m parameter values.

Type of wave interaction	Scat. direction	Amplitude (m)	m parameter (GPa)
P-P	ψ_1	1.97e-11	-2240.07
P-P	ψ_2	5.05e-12	-1191.41
P-P	ψ_3	1.11e-11	-2535.70
P-S	ψ_1	3.89e-13	-893.10
P-S	ψ_4	6.06e-13	-1760.13
S-P	ψ_1	1.66e-13	-373.81
S-P	ψ_3	4.25e-13	-484.57
S-P	ψ_4	1.93e-13	-244.73
S-S	ψ_1	5.33e-13	-435.67
S-S	ψ_2	4.96e-13	-481.07
S-S	ψ_4	4.19e-13	-491.27
m average			-418.52
Standard deviation			87.62

Where the m parameter average and the standard deviation have been calculated deleting atypical values which have been strikethrough at Table 6. Regarding obtained m values it could be observed how values obtained from test in which minor excitation frequency values is a P-wave the parameter take atypical values.

Stobbe [12] and Muir [11] published checked studies in which Murnaghan parameters are analyzed has been selected in order to compare obtained results with existing bibliography values:

$$\begin{aligned} \text{Stobbe (2005) 'm' value: } & -325.0 \text{ GPa} \\ \text{Muir (2009) 'm' value: } & -293.6 \text{ GPa} \\ \text{Nonlinear mixing 'm' value: } & -418.0 \pm 87.6 \text{ GPa} \end{aligned}$$

Obtained result take similar value to existing proposed m parameter values even though to the short sample size analyzed, so being validated that the nonlinear mixing technique could be used as a valid technique to study the nonlinear ultrasonic parameters.

6 Conclusions

Experimental design has been proposed at this paper in order to materialize noncollinear mixing tests for obtaining nonlinear constitutive TOECs. Four aluminum specimens has been designed in order to carry out P-wave – P-wave interaction, P-wave – S-wave interaction, S-wave – P-wave interaction and S-wave – S-wave interaction applying physics fundamentals about wave mode conversion, Zoeppritz wave transmission, and Korneev interaction model.

Correct samples design has been validated making measurements around full samples near field, registering signals only at calculated scattered waves paths.



Murnaghan m parameter values have been obtained for the four type of waves interaction in order to characterize aluminum nonlinear mechanic. Obtained values are coincident with the checked existed values but mayor test number is necessary to get better results accuracy.

The noncollinear mixing technique is validated as a useful technique, which deletes induced tests equipment nonlinearity. Also, the cited experimental technique offers a variant to the classic currently tests, making possible a new vision to the classic nonlinearity study due to it make possible to separate solids nonlinearity (which is directly related with S-wave analysis) and viscous nonlinearity (related directly with P-waves analysis).

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