



## Is it possible to tune a drum?

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### **Abstract**

It is well known that the sound produced by string instruments has a well defined pitch. Essentially, this is due to the fact that all the resonance frequencies of the string have integer ratio with the smallest eigenfrequency. However, it is enough to use Ashbaugh-Benguria bound for the ratio of the smallest two eigenfrequencies to conclude that it is impossible to build a drum with a uniform density membrane satisfying harmonic relations on the eigenfrequencies. On the other hand, it is known since the antiquity, that a drum can produce an almost harmonic sound by using different densities, for example adding a plaster to the membrane. This idea is applied in the construction of some Indian drums like the *tabla* or the *mridangam*. In this work we propose a density and shape optimization problem of finding a composite membrane that satisfy approximate harmonic relations of some eigenfrequencies. The problem is solved by a domain decomposition technique applied to the Method of Fundamental Solutions and Hadamard shape derivatives for the optimization of inner and outer boundaries. This method allows to present new configurations of membranes, for example a two-density membrane for which the first 21 eigenfrequencies have approximate six harmonic relations or a three-density membrane for which the first 45 eigenfrequencies have nine harmonic relations, both involving some multiple eigenfrequencies.

**Keywords:** harmonic drum, Dirichlet Laplacian, eigenvalues, shape optimization.

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