OPTIMIZATION OF DYNAMIC VIBRATION ABSORBERS PLACED ON THE TUNNEL INTERIOR SURFACE TO MITIGATE UNDERGROUND RAILWAY-INDUCED VIBRATION

Behshad Noori[1], Víctor Cubría[2], Robert Arcos[2], Arnau Clot[2], Jordi Romeu[2]

[1]AV Ingenieros
(behshad.noori@avingenieros.com)
[2] Universitat Politècnica de Catalunya
(victor.cubria@estudiante.upc.edu, robrt.arcos@upc.edu, arnau.clot@upc.edu, jordi.romeu@upc.edu)

Abstract
In the present study, dynamic vibration absorber (DVA) has been introduced as an innovative countermeasure in the field of railway-induced vibration. The Pipe-in-Pipe (PiP) model and Green's functions for a two-and-a-half dimensional (2.5D) elastodynamic problem in a full- and half-space have been employed to find the response of the tunnel embedded in a homogeneous half-space due to a harmonic load. DVAs have been coupled to the tunnel interior surface. The effectiveness of DVAs depends on the mass, damping ratio, natural frequency and their distribution. Therefore, an optimization algorithm has been developed to find the optimum parameters of the DVAs, which are those that minimize the vibration levels at the ground surface. Evaluating the efficiency of the optimal DVAs shows that they are an efficient countermeasure to mitigate underground railway-induced vibration.

Keywords: railway-induced vibration, pipe-in-pipe, dynamic vibration absorber, optimization.

PACS no. 07.10.Fq, 02.60.Pn, 87.55.de

1 Introduction

Nowadays, fast and easy commuting is one of the most important necessities in our life; it can be satisfied by the use of underground railways. Not only do these provide timely and costly effective transportation but also cause less traffic congestion, air pollution and noise emission. In spite of their healthy, economic and technological benefits, generated vibration propagates through the tunnel and surrounding soil into nearby buildings and can cause annoyance to the building dwellers and structural damage. In order to solve this problem, firstly, models that can predict railway-induced vibration is needed. Then, those can be employed to study the efficiency of different countermeasures.

Numerical models are powerful tools to estimate underground railway-induced vibration with high accuracy despite the structural complexity of the system. However, these methods demand great computational costs. Some of the numerical models can be found in [1, 2].

Another type of models to predict underground railway-induced vibration are empirical models. These are based on using experimental results to obtain a simple decay law of the vibration with distance [3, 4]. Empirical prediction models present easier, cheaper and faster ways for making a prediction of vibration, while neither of them is suitable for parametric studies.

A theoretical point of view results in better understanding of the dynamic behavior of the system, require far less computational time and also gives the possibility of doing parametric studies. One of the most well-established analytical models is the PiP model. Forrest and Hunt [5] developed the PiP model as a 2.5D semi-analytical model in order to compute soil vibration due to underground railway traffic in a full-space. 2.5D models are referred to a three-dimensional (3D) problems which can be decomposed into a set of two-dimensional (2D) models which depend on the wavenumbers along the invariant direction of the 3D model. The soil response obtained with the PiP model is compared with the one obtained using finite element-boundary element (FE-BE) model in the work of Gupta et al. [6]; good agreement in results is found, being the PiP model computationally more efficient. The PiP model has been used by Hussein and Hunt [7] to evaluate vibration countermeasure performance. It has been also employed by Hussein et al. to calculate vibration from underground railways buried in a homogeneous [8] and multi-layered half-space [9].

A number of methods have been proposed to control and attenuate underground railway-induced vibrations. These methods can be classified as: isolating the source, isolating the receiver and interrupting the vibration path.

Vibration isolation of the source, which is based on changing the dynamic behavior of the track through the insertion of resilient elements or additional masses to its structure, is one of the most common practices in control of a railway-induced vibrations and has been studied by different authors [10-12].

In the case of receiver isolation, rubber blocks or steel springs are mounted between the building and its foundations to reduce transmitted vibration to the building. A comprehensive review of base isolation is presented by Talbot and Hunt [13]. The most used solutions are interrupting the vibration path by using vibration isolation screens [14], open and in-filled trenches [15] or a row of piles [16].

One of the most well-known control devices for cutting the vibration path are DVAs or tuned mass dampers (TMDs). DVA presents unique specifications such as easy-to maintain, uncomplicated design, high reliability and excellent performance. The concept of a DVA was outlined by Watts in 1883 [17]. However, the practical design of vibration absorbers was proposed by Frahm [18] in 1911. During the last century, because of the important role of DVA in vibration attenuation, many studies have been conducted to evaluate the performance of the absorbers as a passive, active and semi-active countermeasures [19]. As structures like bridges, wide-spanned floors, tribunes and high-rise buildings, generally, possess low structural frequencies and little damping, many of these structures originally exhibit medium to high vibration levels. Consequently, the application of DVAs is effective in improving their dynamic performance [20]. The application of largest TMD in the world (660 tons) in the second tallest skyscraper in the world at Taipei 101[21], utilization of eight horizontal and fifty vertical DVAs in Millennium bridge in London [22] and the installation of a 140 tones pendulum absorber in Doha sport city tower in Qatar [23] can be named as some of the most prominent and spectacular usage of DVAs. Optimization of the DVA parameters is one of the most important issues in order to lead to the highest levels of DVAs performance in vibration mitigation [24].

According to the vibration isolation performance of DVAs in vibration control of infrastructures, it is expected to be also an efficient abatement solution for railway-induced vibrations. This efficiency has been evaluated in this study. The remainder of the paper is organized as follows. The methodology, based on PiP and 2.5D Green's functions in full- and half-space, used in this paper to calculate vibration from a tunnel embedded in a half-space is explained firstly. Afterwards, it is explained how to couple DVAs to the tunnel interior surface in order to make a coupled DVA-tunnel-soil system in a half-space. Then, a genetic algorithm (GA) is employed in order to find the optimum parameters of the DVAs, which are those that minimize power spectral density (PSD) on the surface of the ground. Finally, the efficiency of DVAs is evaluated by defining the insertion loss (IL) in PSD due to inserting DVAs.

2 Vibration from a tunnel embedded in a half-space

In the following a methodology is presented that allows to calculate vibration from a tunnel embedded in a homogeneous half-space. In this methodology, it is assumed that near-field displacements of the tunnel are not influenced by the existence of the free surface. It consists of the three following steps that are illustrated in Figure 1:

1) Calculating tunnel-soil interface displacements due to the harmonic point load applied on the tunnel invert by use of PiP model in full-space [5].
2) Employing 2.5D Green's function for a homogeneous full-space [25] to obtain a set of equivalent forces that can produce the same tunnel-soil interface displacements.
3) Calculating the surface response by multiplying 2.5D Green's functions for a homogeneous half-space [26] by the previously obtained equivalent forces.

The PiP model, as the name indicates, can be understood as the coupling of two pipe structures. The inner and the outer pipes, with outer radios being set to infinite, represent the tunnel wall and the surrounding boundless soil with cylindrical cavity, respectively. The former is modelled by using thin shell theory and the soil is modeled as a linear elastic solid. The coupled problem is solved in the wavenumber-frequency domain. To solve the tunnel-soil system completely, three sets of boundary conditions are considered. First, applied load and the stresses on the inside of the tunnel are equal. Second, the displacements must be compatible at the interface of the tunnel and the soil. Finally, the Sommerfeld radiation condition should be satisfied.

In the second step, the tunnel-soil interface displacements are replaced by a set of equivalent forces that can produced these displacements. Green's functions for 2.5D elastodynamic problem in a full-space, developed by Tadeu and Kausel [25], are employed to calculate those forces. In their work, firstly, the method of potentials has been employed to find the response of an infinite, homogeneous space subjected to a line load. Later, by applying radiation condition (or no source at infinity), equilibrium conditions, integrating the stresses in the vicinity of the source and equating their resultant to the applied load, the potentials can be determined. Green's functions can be obtained from the well-known relations between potentials and displacements.

Multiplying these equivalent forces by the 2.5D Green's functions for a homogeneous half-space, developed by Tadeu et al. [26], result in the surface response of the homogeneous half-space due to a unitary harmonic force applied at the tunnel invert. Green's functions for 2.5D elastodynamic problems in a homogeneous half-space were presented subsequent to Green's functions for 2.5D elastodynamic problem in full-space. Imaginary sources along the surface line has been considered in order to provide the free surface. The summation of Green's functions for 2.5D homogeneous full-space and motions resulting from imaginary sources lead to Green's functions for 2.5D homogeneous half-space.

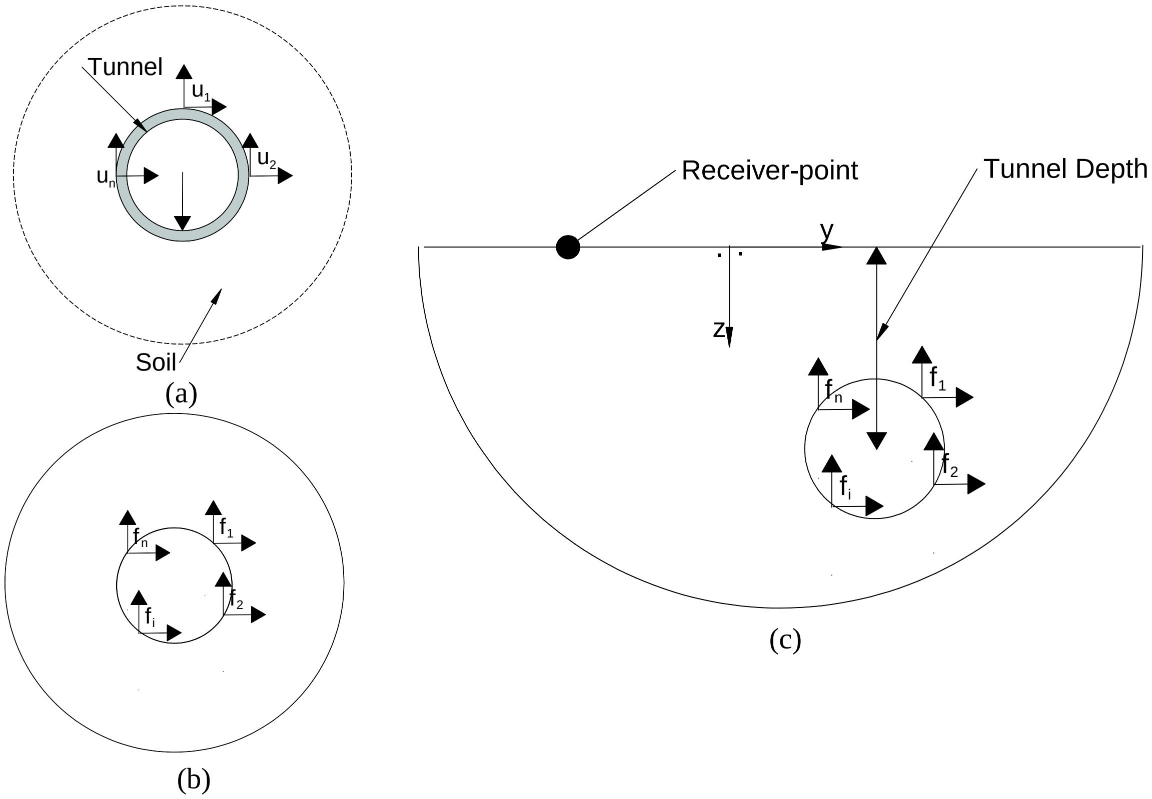


Figure 1 – Three steps to find the surface response; (a) tunnel-soil interface displacements, (b) calculating equivalent forces, (c) calculating surface response due to equivalent forces.

3 Coupling DVAs to the tunnel-soil model

In this section, it is explained how to coupled DVAs to the tunnel which is embedded in a full-space. Initially, in order to achieve a better understanding of the problem, only one periodic distribution of DVAs along the track direction is considered (see Figure 2), so the applied forces to the system is

$$f(x,t) = p(t)\delta(x) + \sum_{n=1}^{+\infty} \left(k_n + c_n \frac{\partial}{\partial t}\right)\left(u_n^{\mathrm{DVA}} - u_n\right)\delta(x - x_n), \tag{1}$$

where the first and second terms in the right-hand side of the equation are the external force and the summation of the DVAs' forces, respectively. k_n and c_n are the stiffness and viscous damping of the n^{th} DVA, respectively. $x_n = nL$, L is the distance between two consecutive DVA, u_n is the displacement of the system at the position and direction of the n^{th} DVA, and u_n^{DVA} is the displacement of center of mass of the n^{th} absorber. The dynamical response of each DVA represented as

$$-\left(k_n + c_n \frac{\partial}{\partial t}\right)\left(u_n^{\mathrm{DVA}} - u_n\right) = m_n \ddot{u}_n^{\mathrm{DVA}}. \tag{2}$$

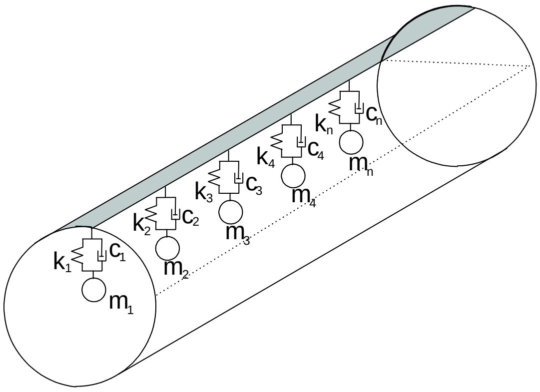


Figure 2 – One longitudinal periodic distribution of DVAs.

The Fourier transform has been defined as

$$\bar{F}(k_x, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,t)\, e^{-i(\omega t - k_x x)} dx dt, \tag{3}$$

where upper line and capital letters denote that the variables are defined in the wavenumber and frequency domain, respectively. This Fourier transform has been applied on both Eqs. (1) and (2). So, displacement of center of mass of the n^{th} absorber in wavenumber-frequency domain can be found as

$$U_n^{\mathrm{DVA}} = \frac{k_n + i\omega c_n}{k_n + i\omega c_n - m_n \omega^2} U_n. \tag{4}$$

By replacing the U_n^{DVA} in the transformed form of the Eq. (1) with its equivalent from Eq. (4), the applied forces to the system, Eq. (1), can be rewritten in the wavenumber-frequency domain as

$$\bar{F} = P + \sum_{n=1}^{+\infty} k'_n (\bar{U}_n)\, e^{i k_x x_n}, \tag{5}$$

where

$$k'_n = (k_n + i\omega c_n)\left(\frac{k_n + i\omega c_n}{k_n + i\omega c_n - m_n \omega^2} - 1\right). \tag{6}$$

As the forces applied to the system are defined in the wavenumber-frequency domain, Eq. (5), by multiplying them to their associated receptances, response at the DVAs' positions can be written as

$$U_b(k_x, \omega) = H(k_x, \omega) + \sum_{n=1}^{+\infty} H_n(k_x, \omega) k'_n U_n(k_x, \omega)\,, \qquad b = 1,2, \ldots, N. \tag{7}$$

By approximating the summation with finite number of DVAs, and applying one inverse Fourier transform, the response at the DVAs' positions can be written as

$$U_b(x_b, \omega) = H(x_b, \omega) + \sum_{n=1}^{N} H_n(x_b - x_n, \omega) k'_n U_n\,, \qquad b = 1,2, \ldots, N, \tag{8}$$

where N is the number of the DVAs along the tunnel, which will be defined in the optimization process. H and H_n are the response at the DVAs' positions due to a unitary harmonic load at the position of the external force and the n^{th} DVA, respectively. According to the symmetry of the tunnel-soil model, H_n can be found by rearranging the H. By computing these receptances, using the PiP methodology, the only remain unknowns are displacements at the DVAs' position. They can be found by solving a system of equations derived from Eq. (8). It can be extended for the case of more than one DVAs' distribution

$$U_{sl} = H(x_{sl}, \omega) + \sum_{p=1}^{P} \sum_{q=1}^{Q} H_p(x_{sl} - x_{pq}, \omega) k'_{pq} U_{pq}\,, \qquad s = 1,2, \dots, P;\; l = 1,2, \dots, q. \qquad (9)$$

P and Q are the number of the DVAs around the circumference and along the longitudinal direction of the tunnel. The position of the DVAs are defined by their subscripts. The summation of the surface response due to unitary harmonic force and DVAs' forces, calculated by following procedure in section 2 is surface response of the DVA-tunnel-soil system due to unitary harmonic force.

4 Optimization of DVAs

The aim of this section is developing an optimization algorithm with the purpose of finding the parameters of the DVAs that minimize the PSD at a receiver point on the surface. In order to solve the optimization problem a GA using MATLAB, in which the design variables are the DVAs parameters with lower and upper bounds constraints, will be implemented. Furthermore, parallel processing has been employed to speed up the optimization algorithm.

The fitness function that GA minimize in this work is the computation of the PSD of the surface response to the applied harmonic point load. This function is computed as follows:

1) Defining the design variables such as the number of the DVAs around the circumference of the tunnel and along the invariant direction of tunnel, their positions, stiffness, damping and masses. These data are managed as an input for the second function. The lower and upper bounds constrains for mass are 500 and 1000 kg, respectively; and for stiffness are 5 and 50 MN m-^{1}. The damping is considered as 1% of the stiffness and it is not defined as an independent design variable. The minimum distance between consecutive DVA is considered as one meter. Noteworthy, the maximum of six DVAs along the circumference of the tunnel and maximum of forty DVAs for each longitudinal distribution are taken into account.

2) Calculating the DVAs' forces by use of Eq. (9), considering the characteristics of the DVAs chosen in the previous step. The receptances due to the force applied at the positions of the external force, H, and all possible positions of the DVAs' forces, H_n, are calculated just once and before calling the GA to decrease the computational costs.

3) Calculating the surface response of the tunnel-soil system due to unitary harmonic force in the presence of the DVAs in space-frequency domain by using methodology explained in section 2. The final output is a 3D matrix like $[N_\omega, N_x, 3N_\mathrm{r}]$.The first one represents the number of the frequencies, the second one is the number of the points for sampling along the x direction and in the last one represents the number of the receiver-point.

4) As human exposure to vibration is different in compare to structural vibration, minimizing the vibration level on the surface may not lead to more comfort for the occupants. Therefore, the normative that evaluate the human exposure to the whole-body vibration [27] is applied to the response of the complete system calculated in the third function.

5) Finally, PSD is calculated as the final output of the fitness function by using

$$\mathrm{PSD} = \frac{1}{M^2} \sum_{m=1}^{M} A_m^2 W_m^2 \qquad (10)$$

$$A^2 = \sum_{j=1}^{N_x} \left(\left|\ddot{X}_j\right|^2 + \left|\ddot{Y}_j\right|^2 + \left|\ddot{Z}_j\right|^2 \right). \tag{11}$$

where W_m is the frequency weighting for the acceleration as the input quantity, the summation has been performed over the frequency; and X, Y and Z are the surface response components.

5 Results and discussion

In this section, the efficiency of DVAs in minimizing the PSD due to unitary harmonic force, taking into account the human-body exposure, are evaluated. First, by employing PiP model, the tunnel-soil interface displacements due to the unitary external force is evaluated at forty points around the circumference of the tunnel, ($n = 40$ in Figure.1). This number of the points is chosen according to the other studies in which PiP are employed [8, 9]. Then, by using 2.5D Greens' functions for full- and half-space, the response at the receiver-point, $(x_r, y_r, z_r) = (0, -10, 0)$ (m), is calculated in wavenumber-frequency domain. The surface responses of the system due to the unitary forces at the DVAs' possible positions, which are those forty points, are calculated by employing the same procedure. DVA's possible positions have been considered as the same as those in which the displacements have been computed just to reduce the computational costs.

The values presented in Table 1 are considered to model the soil, where E, υ and ρ are Young's modulus, Poissons ratio and density of the soil, respectively. D_p and D_s are the hysteretic damping ratios for P- and S-waves. In Table 2, the parameters used to model the tunnel are presented. $E_t, \upsilon_t, \rho_t, h$ and r are Young's modulus, Poissons ratio, density, thickness and middle radius of the tunnel, respectively. The position of the tunnel center is defined as $(y_t, z_t) = (L_t, D_t)$.

Table 1 – Parameters used to model the soil

Parameters	E (MPa)	ν(-)	ρ (kg m^{-3})	D_p(-)	D_s(-)
Values	366	0.44	2000	0.03	0.03

Table 2 – Parameters used to model the tunnel

Parameters	E_t (GPa)	ν_t(-)	ρ_t (kg m^{-3})	h (m)	r (m)	D_t (m)	L_t (m)
Values	50	0.3	2500	0.3	3	40	10

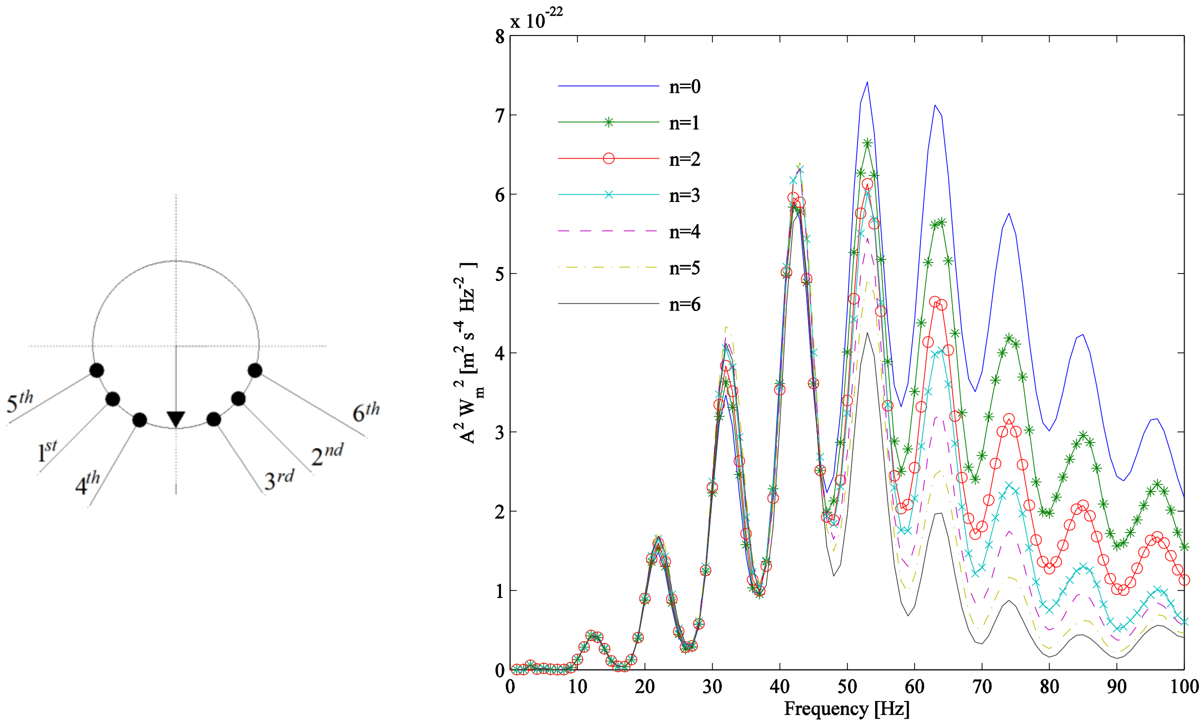
Once the receptances of the coupled DVA-tunnel-soil system are computed, the GA is called to find the optimum parameters of DVAs that can minimized the PSD for the frequency range of [1 100] Hz. In the optimization process, first, only one distribution of DVAs is considered and optimized, then these parameters are used to find the optimum parameters of the second distribution and so on. Note that all DVAs along the same longitudinal periodic distribution have the same properties. Forty DVAs have been considered as the upper bound for longitudinal distribution of DVAs, however, analyzing of the results show that more than six DVAs in each distribution has a very slight effect in vibration mitigation. So, six DVAs are considered in each longitudinal distribution as an upper bound. In Table 3, the optimum parameters for each distribution can be found; the optimum longitudinal distance between two consecutive DVAs in all distributions is one meter. The maximum mass is the most efficient solution, this also has been achieved in optimization. The optimum positions for six distributions of DVAs are close to the external load and are symmetric, as it has expected, and they are shown in Figure 3-a. It is easier to implement a DVA in these positions than in the tunnel ceiling but that the structural stability and practical construction of the DVA are beyond the aim of the present work.

Table 3 – Optimum parameters for each DVAs' distributions shown in Figure 3-a

Distribution	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Stiffnss (MN m^{-1})	50.00	50.00	36.97	49.49	49.74	22.01
Mass (kg)	999	1000	998	993	1000	992
y_{DVA} (m)	7.15	12.85	12.67	7.33	7.04	12.96
z_{DVA} (m)	40.93	40.93	41.36	41.36	40.46	40.46

After finding the optimum parameters of the DVAs, Eq. (11) and the frequency weighting have been used to calculate $A^2(W_m)^2$, without DVAs and in the presence of the n distributions of the DVAs, where $n = 1,2,\ldots,6$, for the frequency range of [1 100] Hz. The results are plotted versus frequency in Figure 3-b, in which, for example, $n = 3$ means that first, second and third distribution of DVAs are considered. The insertion loss (IL) is also defined to evaluate the loss of the PSD, resulting from the insertion of DVAs, as

$$\mathrm{IL} = 10\log_{10}\left(\frac{\mathrm{PSD}_0}{\mathrm{PSD}_n}\right). \qquad (12)$$



(a) (b)

Figure 3: (a) optimized DVAs' positions around the circumference of the tunnel; (b) $A^2(W_m)^2$ calculated using Eq. (11) and frequency weighting; and n is the number of the DVAs' distributions.

PSD_0 is the power spectral density for the system without DVAs and PSD_n for the system with n distributions of the DVAs.The results are shown in Table 4. As can be seen, by using only one distribution of 6 DVAs the IL of 0.9 dB can be achieved, and it can be arrived to 3.8 dB by using 6 distributions of 6 DVAs, or in total 36 DVAs.

Table 4 – IL due to using n distributions of the DVAs

Number of DVAs distributions	1	2	3	4	5	6
IL (dB)	0.9	1.6	2.1	2.7	3.1	3.8

6 Conclusion

In the present paper, the efficiency of DVAs as an innovative countermeasure in the field of underground railway-induced vibration control has been studied. A well-stablished semi-analytical models, the PiP model, has been employed to find the surface response of the tunnel embedded in a half-space in the wavenumber-frequency domain. Then, the DVAs have been coupled to the tunnel interior surface in the space-frequency domain to make a coupled DVA-tunnel-soil system. The surface response of the complete model due to the unitary forces at the DVAs' positions have been calculated. The GA has been used to find the optimum parameters of the DVAs such as their stiffness, mass and positions in order to minimize PSD which is defined as the fitness function. In order to evaluate the efficiency of the DVAs, an insertion loss factor has been defined. According to the results, DVAs can be presented as an effective countermeasure in vibration control of underground railway-induced vibration as an IL of 0.9 dB has been achieved by using only 6 DVAs. This can be increased to 3.8 dB by inserting 36 DVAs separated over 6 different distributions.

Acknowledgements

This work has been carried out in the context of the Industrial Doctorates Plan, managed with financial support from AGAUR - Generalitat de Catalunya and AV Ingenieros, developed in a partnership with Polytechnic University of Catalunya (UPC). The authors would like to extend their thanks to ISIBUR project: *"Innovative Solutions for the Isolation of Buildings from Underground Railway-induced vibrations"* funded by the Spanish Government - (TRA2014-52718-R).

References

[1] Andersen L.; Jones C. J. C. Coupled boundary and finite element analysis of vibration from railway tunnels-a comparison of two- and three-dimensional models, *Journal of sound and vibration*, Vol. 293 (3), 2006, pp. 611–625.

[2] Galvín, P.; François, S.; Schevenels, M.; Bongini, E.; Degrande, G.; Lombaert, G. A. 2.5D coupled FE-BE model for the prediction of railway induced vibrations, *Soil Dynamics and Earthquake Engineering*, Vol. 30 (12), 2010, pp. 1500–1512.

[3] Hanson, C. E.; Ross, J. C.; Towers, D. A. High-Speed Ground Transportation Noise and Vibration Impact Assessment | Federal Railroad Administration, 2005.

[4] Lombaert, G.; François, S.; Verbraken, H.; Degrande, G; Thompson, D. Numerical, experimental and hybrid methods for the prediction of railway-induced ground vibration, *in Proceedings of the 9th International Conference on Structural Dynamics, EURODYN 2014*, 2014, pp 91–99.

[5] Forrest, J. A.; Hunt, H. E. A three-dimensional tunnel model for calculation of train-induced ground vibration, *Journal of sound and vibration*, Vol. 294 (4-5), 2006, pp. 678–705.

[6] Gupta, S.; Hussein, M. F. M.; Degrande, G.; Hunt, H. E. M.; Clouteau, D. A comparison of two

numerical models for the prediction of vibrations from underground railway traffic, *Soil Dynamics and Earthquake Engineering*, Vol. 27 (7), 2007, pp. 608–624.

[7] Hussein, M.; Hunt, H. E. M. A power flow method for evaluating vibration from underground railways, *Journal of sound and vibration*, Vol. 293 (3-5), 2006, pp. 667–679.

[8] Hussein, M. F. M.; Gupta, S.; Hunt, H. E. M.; Degrande, G.; Talbot, J. P. An efficient model for calculating vibration from a railway tunnel buried in a half-space, *in Proceedings of the Thriteenth International Congress on Sound and Vibration, Vienna*, 2006.

[9] Hussein, M. F. M.; François, S.; Schevenels, M.; Hunt, H. E. M.; Talbot, J. P.; Degrande, G. The fictitious force method for efficient calculation of vibration from a tunnel embedded in a multi-layered half-space, *Journal of sound and vibration*, Vol. 333 (25), 2014, pp. 6996–7018.

[10] Costa, P. A.; Calcada, R.; Cardoso, A. Influence of train dynamic modelling strategy on the prediction of track-ground vibrations induced by railway traffic, *Proceedings of the Institution of Mechanical Engineers, Journal of Rail and Rapid Transit*, Vol. 226 (4), 2012, pp. 434–450.

[11] Witt, S. The influence of under sleeper pads on railway track dynamics, PhD thesis, Linköping University, 2008.

[12] Costa, P.; Calçada, R.; Cardoso, A. S. Ballast mats for the reduction of railway traffic vibrations. Numerical study, *Soil Dynamics and Earthquake Engineering*, Vol. 42 (1), 2012, pp. 137–150.

[13] Talbot J.; Hunt H. E. M;, Isolation of Buildings from Rail-Tunnel Vibration: a Review, *Building Acoustics*, Vol. 10 (3), 2003, pp. 177–192.

[14] François, S.; Schevenels, M.; Thyssen, B.; Borgions, J.; Degrande, G. Design and efficiency of a composite vibration isolating screen in soil, *Soil Dynamics and Earthquake Engineering*, Vol. 39 (1), 2012, pp. 113–127.

[15] Andersen L.; Nielsen, S. Reduction of ground vibration by means of barriers or soil improvement along a railway, *Soil Dynamics and Earthquake Engineering*, Vol. 25 (7), 2005, pp. 701–716

[16] Kattis, S.; Polyzos, D.; Beskos, D. Modelling of pile wave barriers by effective trenches and their screening effectiveness, *Soil Dynamics and Earthquake Engineering*, Vol. 18b (1), 1999, pp. 1–10.

[17] Watts, P. On a method of reducing the rolling of ships at sea, *The Royal Institution of Naval Architects*, Vol. 24, 1883, pp. 165–190.

[18] Frahm, H. Device for damping vibration of bodies,*U.S. Patent. No. 989958*, 1911.

[19] Sun, J.; Jolly, M.; and Norris, M. Passive, Adaptive and Active Tuned Vibration Absorbers—A Survey, *Journal of Mechanical Design*, Vol. 117 (B), 1995, pp 234.

[20] Carpineto, N.; Lacarbonara, W.; Vestroni, F. Mitigation of Pedestrian-induced Vibrations in Suspension Footbridges via Multiple Tuned Mass Dampers, *Journal of Vibration and Control* Vol. 16 (5), 2010, pp. 749–776.

[21] Kourakis, I. Structural systems and tuned mass dampers of super-tall buildings : case study of Taipei 101, Massachusetts Institute of Technology, 2007.

[22] Newland, D. E. Vibration of the London Millennium Bridge: cause and cure, *International Journal of Acoustics and Vibrations*, Vol. 8 (1), 2003, pp. 9–14.

[23] Nawrotzki, P. Tuned-mass systems for the dynamic upgrade, *Eleven East Asia Pacific Conference Structural Engineering Construction*, 2008, pp. 1–9.

[24] Noori B.; Farshidianfar, A. Optimum design of dynamic vibration absorbers for a beam, based on H_∞ and H_2 optimization, *Archive of Applied Mechanics*, Vol. 83 (12), 2013, pp. 1773–1787.

[25] Tadeu A.; Kausel, E. Green's functions for two-and-a-half-dimensional elastodynamic problems, *Journal of Engineering Mechanics*, Vol. 126 (10), 2000, pp. 1093–1097.

[26] Tadeu, A.; António, J.; Godinho, L. Green's function for 2.5D elastodynamic problems in a half-space, *Computational Mechanics*, vol. 27 (6), 2001, pp. 484–491.

[27] ISO 2631-2:2003 Mechanical vibration and shock, *Evaluation of human exposure to whole-body vibration, Part 2: Vibration in buildings (1 Hz to 80 Hz)*, 2003.