From Quasi-Perfect to Broadband Sound Diffusion Using Metadiffusers

Eric Ballestero[1], Noé Jiménez[2], Jean-Philippe Groby[3], Haydar Aygun[1], Stephen Dance[1] and Vicent Romero-García[3]

[1]The Acoustics Group, London South Bank University, 103 Borough Road, London SE1 0AA, United Kingdom.

[2]Instituto de Instrumentación para Imagen Molecular, Consejo Superior de Investigaciones Científicas (CSIC), Universitat Politècnica de València, Camino de Vera s/n, 46022 València, Spain.

[3]Laboratoire d'Acoustique de l'Université du Mans (LAUM), UMR CNRS, 6613, Institut d'Acoustique - Graduate School (IA-GS), CNRS, Le Mans Unversité, Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France.

Abstract

Surfaces that display omnidirectional sound diffusion may prove impractical to design under traditional means, mostly due to the considerable bulkiness and lack of adaptability of typical quarter-wavelength diffusing strategies. Thus, designing ultra-thin sound diffusers displaying near perfect diffusion values can become quite a challenge. In this work, we then propose a method for obtaining quasi-perfect and broadband sound diffusion coefficients using deep-subwavelength acoustic diffusers, i.e., metadiffusers. Analytical and numerical results show that such aim can be reached with panels 1/30th thinner than traditional quarter-wavelength sound diffusers, spanning from quasi-perfect values for frequency ranges 1/3 of an octave to average diffusion values of 0.8 over 2.5 octaves. This work demonstrates the effectiveness and versatility of metadiffusers to tailor the nature of their diffuse field and their potential for practical situations where they can outshine classical solutions.

Keywords: sound scattering & diffusion, metamaterials, slow sound structures, optimization

1 Introduction

Sound diffusion is based on the physical principles of wave diffraction. Such phenomenon can be fundamentally described through the Helmholtz-Kirchoff integral formulation which relates the output scattered wave field generated by the diffracting surface to a certain input wave field [1]. In acoustics, surfaces that scatter waves in such manner are commonly referred to as sound diffusers; the purpose of which is to disperse sound evenly or focus it into specific directions. In this work, a particular emphasis is given to the former goal, and more particularly to sound diffusers that can radiate waves omnidirectionally, resulting in a perfect diffusion of the reflected sound field.

Sound diffusers achieve their goal of dispersing sound through a specific distribution of the acoustic impedance along their surface. Traditionally, these locally-reacting surfaces are made by juxtaposing quarter-wavelength resonators (QWRs) of different depths, such as wells, thus creating a frequency-dependent profile of the reflection coefficient of the surface. This type of phase-grating sound diffusers are usually referred to as Schröeder diffusers and follow a numerical sequence with flat Fourier transform properties in order to scatter sound as evenly as possible [2].

However, this strategy often results in sizeable structures for large wavelengths, as the maximum phase shift of the reflection coefficient occurs at $L = c_0/4f$, where f is the working frequency, L is the depth of the QWR, and c_0 is the speed of sound in air. This size limitation, along the fact that QWRs do not offer a great tunability of the magnitude and phase of the reflection coefficient limit the use of Schroeder diffusers in situations where a greater adaptability is required for the magnitude and phase behaviour of the structure.

2 Metadiffusers: deep-subwavelength sound diffusers

Through the last decades, various solutions have been proposed in order to palliate with the bulkiness of Schröeder diffusers for low frequencies. Lately, metamaterial-inspired strategies for designing metasurfaces presenting simultaneously efficient diffusion properties and deep-subwavelength dimensions (i.e., dimensions much smaller than the working wavelength) have started to be proposed as well [3]. Here, this study focuses on the concept of metadiffusers [4].

Metadiffusers are rigidly backed slotted panels based on slow-sound metamaterials, where each slit is loaded by an array of Helmholtz resonators (HRs). That way, strong dispersion is introduced and the effective sound speed inside each slit is drastically reduced in the low frequency regime. This causes the quarter-wavelength resonance to be shifted into the deep-subwavelength regime, therefore strongly reducing the effective thickness of the panel. The geometry variables of each metadiffuser design can be calculated following an optimization procedure for replicating the reflection coefficient profile corresponding to a given numerical sequence at a particular frequency. As such, many designs were proposed based on numerical sequences that already exist for traditional diffusers. Recently, the diffusion characteristics of a 3D-printed quadratic residue metadiffuser (QRM) were experimentally measured and compared to analytical and numerical results [5], where good agreement was found.

However, the potential for metadiffusers to present omnidirectional sound scattering, referred here as perfect diffusion, has yet to be evaluated and discussed. That is why we present in this work a method for maximizing quasi-perfect diffusion (QPD) of sound in both narrow and broad frequency bands based on metadiffusers.

The scattered sound pressure distribution over a reflecting surface can be usually described by a Fourier transform of the complex reflection coefficient. In such case, following Fraunhofer's diffraction theory as an approximation of the Helmholtz-Kirchhoff integral equation, the far-field scattered pressure distribution at normal incidence, $p_s(\theta,\phi)$, of a reflecting rectangular surface with a spatially-dependent reflection coefficient, $R(x,y)$, of size $2a$ and $2b$ in the x and y directions, respectively, can be calculated using the Fraunhofer integral

$$p_s(\theta,\phi) = \int_{-a}^{a}\int_{-b}^{b} R\,(x,y) e^{ik(x\sin\theta\sin\phi + y\sin\theta\cos\phi)} \mathrm{d}x\mathrm{d}y$$

where θ and ϕ are the elevational and azimuthal angles, respectively.

The diffusion coefficient of a surface rates the uniformity of the aforementioned scattered sound field. The directional diffusion coefficient, δ_ψ, produced when a sound diffuser is illuminated by a plane wave at incident angle $\psi = (\theta', \phi')$ (primed superscripts denoting incident angles) can be estimated from the hemispherical distribution

$$\delta_\psi = \frac{([\, I_s\,(\theta,\phi) dS]^2 - [\, I_s^2\,(\theta,\phi) dS])}{(\, I_s^2\,(\theta,\phi) dS)}$$

where $I_s(\theta,\phi) \propto |p_s(\theta,\phi)|^2$ is proportional to the scattered intensity. The integration is performed over a hemispherical surface $(-\pi/2 \leq \theta \leq \pi/2)$ and $(0 \leq \phi \leq 2\pi)$, where $\mathrm{d}S = \mathrm{d}\theta\mathrm{d}\phi$. This coefficient must be normalized to that of a plane reflector, δ_{flat}, so as to eliminate the diffracting effect caused by the finite size of the structure, i.e, $\delta_{n,\psi} = \left(\delta_\psi - \delta_{flat}/1 - \delta_{flat}\right)$. We analyse here the 2D case of a normal plane wave incidence in the elevational plane only, i.e., $\delta_{n,\psi} \equiv \delta_0$.

The analytical surface reflection coefficients required to calculate the far-field sound scattering were obtained with the Transfer Matrix Method (TMM) [4], which relates the acoustic pressures and normal particle velocities at the extremities of a one-dimensional acoustic system. Numerical simulations were computed using a Finite Element Method (FEM) in COMSOL Multiphysics®. The surface was installed at the centre of a circular domain filled with air surrounded by a concentric perfectly matched layer (PML) with a far-field boundary condition at the boundary of the air domain to simulate the radiation condition.

3 Quasi-perfect diffusion (QPD)

Previous metadiffuser designs were optimizing the geometry of the metadiffuser in order to produce a a reflection coefficient profile fitting a particular numerical sequence at a certain frequency. Here, a different approach is proposed in order to reach QPD. Instead, we look for the geometry of the metadiffuser that directly maximizes the normalized diffusion coefficient at the target frequency. The optimization uses a constrained minimization algorithm where the cost function $\epsilon = 1 - \delta_{n,\psi}(f)$ is minimized so that the normalized diffusion coefficient would tend to unity at the target frequency. A metadiffuser being composed of N slit unit cells of periodicity $a_x = D/N$, this results in a reflection coefficient profile along the x direction with N different values over the total length of the surface. By optimizing the geometry of such metadiffuser, i.e., the widths and lengths of each slit and its constituting HRs, a phase profile for reaching QPD can thus be obtained with the aforementioned methodology.

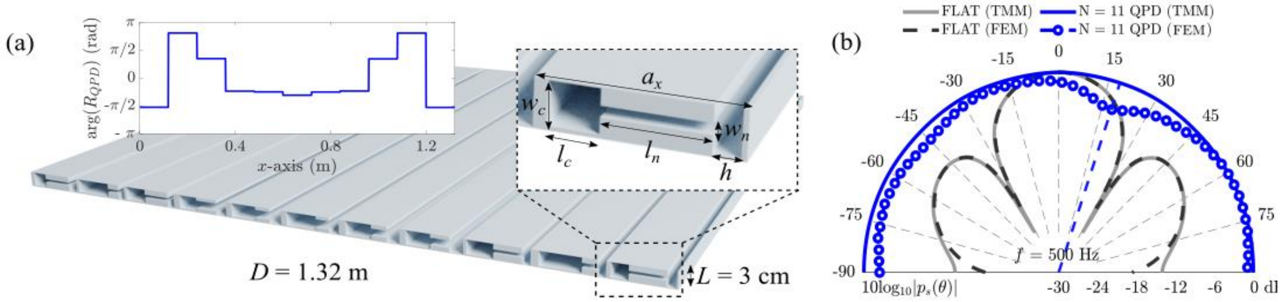

Figure 1: (a) Geometry of a quasi-perfect diffusion (QPD) metadiffuser design made of 11 slits. (inset) Target phase of the reflection coefficient at 500 Hz. (b Far field scattered sound energy at f = 500 Hz for QPD-metadiffuser and a flat reference surface of the same width

Figure 1(a) shows the geometry obtained from the optimization of an N = 11 QPD-metadiffuser, with M = 1 HR, lateral dimension D = 1.32 m, and height L = 3 cm, as well as the target phase of the reflection coefficient required for QPD at 500 Hz. The spatial Fourier transform of such a phase profile provides a constant scattered amplitude in space. A large panel width D = 1.32 m has been taken here in order to avoid the low frequency diffraction regime that would otherwise occur at dimensions close to $k \approx$ 0.7m at 500 Hz.

Figure 1(b) displays the scattered far-field for the same QPD-metadiffuser, where the magnitude of the normalized scattering obtained numerically at $\theta \approx 17^o$ is -4 dB. The numerical polar distribution at other angles is otherwise quite uniform, resulting in high, quasi-perfect sound diffusion performance of $\delta_{n,0} = 0.92$, as opposed to the analytical value of $\delta_{n,0} = 0.99$.

4 From QPD to highly efficient broadband sound diffusion

Starting from the optimization paradigm established for QPD, a more extended optimization scenario can be made for broadband sound diffusion performance. In such a case, broadband metadiffusers are designed by modifying the optimization cost function to account for normalized diffusion coefficients held between a low and high cut frequencies, i.e., $\epsilon = 1 - \delta_{n,0,avge}(n_f)$ where

$$\delta_{n,0,avge}(n_f) = \int_{f_{low}}^{f_{high}} \delta_{n,0}\, df / n_f$$

is the normalized diffusion coefficient averaged over n_f frequency samples. Thus, highly efficient broadband metadiffusers can be designed where the geometry constraints of the metadiffuser remain the same as

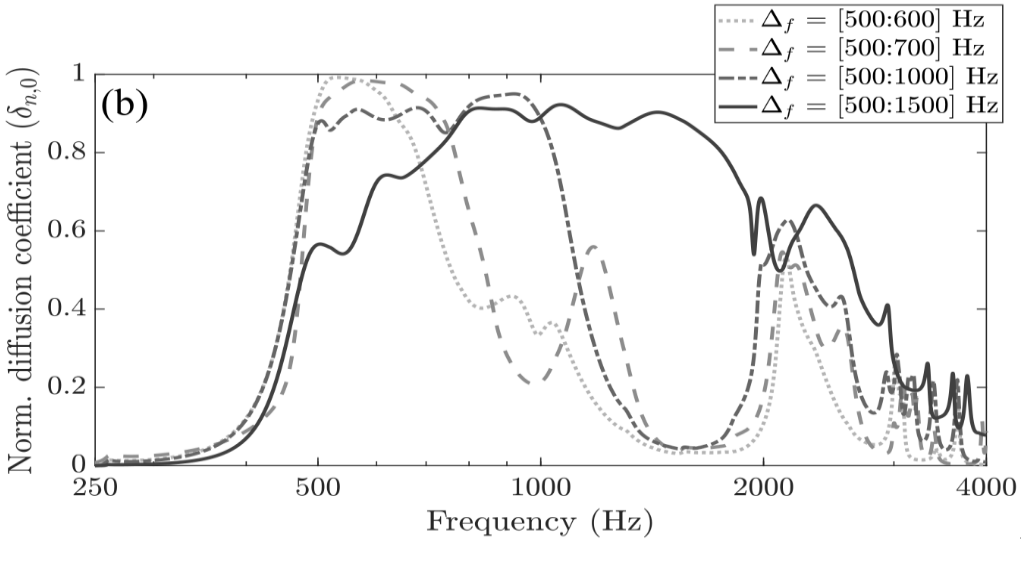

Figure 2: Normalized diffusion coefficients of various broadband metadiffuser designs for different frequency ranges.

previously while the rest of the geometry (slits and HRs) would now fit the cost function for several frequency ranges.

Figure 2 shows examples of the various normalized diffusion coefficients obtained through different $\Delta_f = [f_{low}\!:\!f_{high}]$ frequency bandwidths tested. In the case of $\Delta_f = [500\!:\!600]$ Hz, a value $\delta_{n,0,avge} = 0.98$ is achieved. This is an extremely high value considering its frequency spans over 100 Hz. For the other cases, one can observe that as the frequency range Δ_f increases, $\delta_{n,0,avge}$ decreases. Yet, the latter still remains at high values, viz., 0.96, 0.90, and 0.84, for broader frequency ranges. In addition, a diffusion peak can be outlined for all the Δ_f designs. In the case of $\Delta_f = [500\!:\!1500]$ Hz, this peak provides a continuous decrease in the normalized diffusion coefficient to even higher frequencies.

5 Conclusion

In this work, we have demonstrated the potential of metadiffusers for displaying quasi-perfect normalized sound diffusion coefficients within deep-subwavelength dimensions. The ability to obtain such range of efficient scattered sound distributions within ultra-thin dimensions, instead of larger alternatives, can be welcome when dealing with environments where space is at a premium, e.g., aerospace applications or orchestra pits. The results shown in this work demonstrate the usefulness of metadiffusers to be applied in many practical situations, where they can outshine classical solutions due to their versatility. For more information concerning this work, please refer to the published material in Ref. [6].

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