ABSTRACT
Nonlinearity parameter can be used for the estimation of the gas void fraction in gassy sand sediment. If two primary acoustic waves of different frequencies are incident on gassy sediment, nonlinear acoustic waves can be strongly generated at the difference frequency of the primary frequencies. Nonlinearity parameter of gassy sediment is related to the gas void fraction provided the primary frequencies are lower than bubble resonance frequencies. In the present study, the difference frequency wave was employed to estimate nonlinearity parameter of gassy sand sediment under laboratory conditions. Nonlinearity parameter in gassy sand sediment was \( e = 2.54 \times 10^3 \). Then, the estimated gas void fraction in gassy sand sediment was \( \beta = 7.98 \times 10^{-4} \). This value well agreed with that estimated from the sound speed variation. This study suggests that the difference frequency acoustic wave method seems very feasible to estimate the void fraction in gassy sand sediment.

INTRODUCTION
The porous media, such as soils, rocks, and sediments, exhibit high nonlinearity in comparison with the nonporous media [1, 2]. The nonlinearity of the sediments can play an important role for oil field prospecting and ecological monitoring in the ocean. Most of the sediments in the ocean contain a lot of bubbles, which show very sensitive nonlinear responses for acoustic waves. Therefore, nonlinear acoustic responses of the bubbles can be usefully utilized for estimation of the gas void fraction in gassy sediment [3]. In this study, the nonlinear generation of difference frequency acoustic waves in gassy sand sediment under laboratory conditions was investigated. The nonlinearity parameter of the sediment was evaluated by using a theory of the parametric acoustic array for difference frequency acoustic wave. Gas void fraction of the sediment was estimated from the evaluated nonlinearity parameter. It was also compared with that estimated from the sound speed variation in the sediment.

THEORY
If we consider that two primary acoustic waves are incident on a sediment layer with thickness \( l \) in water as shown in Fig.1(a), the nonlinear acoustic wave at the difference frequency can be generated in the layer. The acoustic pressure fields of two primary acoustic waves in the sediment layer located at far distance \( H \) can be expressed as follows:

\[
P_{1,2} = (A_{1,2}TR_{1,2}D_{1,2}(\theta'')) \exp\{-\alpha_{1,2w}(z-H)\} \exp\{i(\omega_{1,2} - k_{1,2w}(z + p_{l,2}^2 / 2H))\},
\]

\( A_{1,2} \) are pressure amplitudes of two primary waves at the piston source, \( D_{1,2}(\theta') \) and \( R_{1,2f} = \pi a^2 / \lambda_{1,2w} \) are the beam directivity functions and the Rayleigh distance for primary frequencies at the piston source in water, \( T = 2p_jc_j / (p_jc_j + p_oc_w) \) is the transmission coefficient of the primary acoustic wave from water into the sediment layer. Then, the backscattered difference frequency wave field from the layer can be given in forms of
\[
p_d = \frac{e^{\omega_0 T' T^2 A_{T'} R_{T'} R f_1 \exp(\alpha_{d1})}}{4 \rho_p c_p^3 H^2 (2 j k_{dw} + \alpha_{f1})} \left(1 - e^{-(2 j k_{dw} + \alpha_{f1}) H} \right)^2 \left(1 - e^{-\frac{H^2}{H}} \right)
\]

(Eq.2)

Figure 1. Schematic diagrams for (a) theoretical description and (b) experimental setup.

Nonlinearity parameter of the sediment layer, \(e\) in Eq. (2) can be easily determined by

\[
\varepsilon = \frac{-j 4 p_d \rho_p c_p^3 H^2 \exp(-\alpha_{d1})(2 j k_{dw} + \alpha_{f1})}{\alpha_{d1} T' T^2 A_{T'} R_{T'} R f_1 \exp(-\alpha_{d1})(2 j k_{dw} + \alpha_{f1})} \left(1 - \exp(-(2 j k_{dw} + \alpha_{f1}) H) \right)^2 \left(1 - \exp(-\frac{H^2}{H}) \right)^2
\]

(Eq.3)

If the gas bubbles exist in the pore water of the sediment layer and the primary frequencies are lower than resonance frequencies of the bubbles, the gas void fraction \(\beta\) in the sediment layer can be simply expressed with the nonlinearity parameter as in water containing gas bubbles as follows [4]:

\[
\beta = \frac{2 \varepsilon \gamma^2 P_0^2}{\rho_p c_p^4 (\gamma + 1)}
\]

(Eq.4)

where \(\gamma = 1.4\) is the polytropic exponent of the bubble gas, \(P_0 = 10^5\) Pa is static pressure. The gas void fraction \(\beta\) in Eq. (4) can be also expressed using the speed of sound in sediment pore water as follows [4]:

\[
\beta = \frac{(c_u/c_p)^2 - 1}{\rho_p c_u^4 c_p^{4/3}} \gamma \rho u
\]

(Eq.5)

where \(c_p\) is the speed of sound in sediment pore water with gas bubbles.

**EXPERIMENTAL SETUP**

The water-saturated sand sediment layer for experimental measurements was prepared under laboratory conditions. The porosity and the density of the sediment layer were 41% and 1956 kg/m³, respectively. The sediment layer was packed into a Lucite box with a volume of 300×300×50 mm³ containing the gas bubbles produced through electrolysis-type bubble maker in water. The main sizes of the gas bubbles measured in water were distributed around the radius of 20 \(\mu\)m with resonance frequency of 165 kHz.

A schematic diagram of the experimental setup for acoustic measurements is shown in Fig. 1(b). The sediment layer was located at far distance of 900 mm from the transducer (RESON TC2122). The diameter of transducer was 180 mm and its center frequency was 33 kHz. The transducer was simultaneously driven at two primary frequencies of 28 and 33 kHz to get the
difference frequency of 5 kHz. The temperature of water tank was maintained at 12 °C. The incident signals on the gassy sand sediment layer at two primary frequencies were tone burst sinusoidal signals with pulse duration of 1 ms. The acoustic pressure amplitude for the primary acoustic wave of 28 kHz at the transducer was 58 kPa. It was 53 kPa for primary acoustic wave of 33 kHz. The reflected signals from the gassy sand sediment layer were received by a hydrophone (B&K 8103). To measure the sound speed and the acoustic attenuation in the sediment layer [5, 6], another hydrophone (EDO 6600) was installed at the distance of 25 mm in forward direction of the sediment layer. The received signals were acquired using a 300-MHz digital storage oscilloscope (LeCroy 9310M) and stored on a computer for off-line analysis.

RESULTS and DISCUSSION

Figure 2 shows time signals received at the hydrophone (EDO 6600) with and without the gassy sand sediment in water at a center frequency, 33 kHz of the transmitter. The arrival time difference between the two signals in Fig. 2 was \( t = 64.7 \mu s \). In this study, the arrival time difference of time signals with and without the water-saturated sand sediment was also measured as \( t' = 5.5 \mu s \). Then, the arrival time difference of time signals with and without the gas bubbles in sediment could be given as \( t_p = t - t' = 70.2 \mu s \). Therefore, the sound speeds in gassy sand sediment and the sediment pore water were estimated \( c_s = 505 \text{ m/s} \) and \( c_p = 478 \text{ m/s} \) [5], respectively. Then, the estimated gas void fraction from Eq. (5) was \( \beta = 5.48 \times 10^{-4} \).

![Figure 2. Time signals received at the hydrophone (EDO 6600) with and without gassy sand sediment in water at a center frequency, 33 kHz of the transmitter.](image)

The waveform of time signal transmitted through gassy sand sediment in Fig. 2 seems to be extended in comparison with that of the reference signal. It can be caused by scattering effects due to the gas bubbles in sediment. This scattering effect makes it difficult to distinguish the original time signal from the transmitted signal through sediment. Therefore, the time signal in the circle zone of Fig. 2 was determined as the direct receiving signal of the original time signal. This time signal was used to estimate the attenuation coefficients of the primary waves [6]. The attenuation coefficients of the primary waves at the frequencies of 28 and 33 kHz were estimated as 57.98 and 71.10 Np/m, respectively.

Figure 3(a) shows the frequency spectrum of the signal received at a distance of 900 mm from the transmitter without gassy sand sediment in water. As shown in Fig. 3(a), the difference frequency component almost appeared below the background noise level. Figure 3(b) shows the frequency spectrum of the signal reflected from gassy sand sediment in water. The pressure levels at the primary frequencies of 28 and 33 kHz in Fig. 3(b) were largely decreased in comparison with those in Fig. 3(a). The decrease of the pressure level at the primary frequency of 28 kHz was about 13 dB. It was about 20 dB for the primary frequency of 33 kHz. The pressure level at the difference frequency of 5 kHz in Fig. 3(b) was 167.9 dB, which was 38 dB
higher than the background noise level. This could be caused by the high nonlinearity of the gassy sand sediment. The nonlinearity parameter of the sediment could be determined from the difference frequency level. It was estimated as $e = 2.54 \times 10^3$. In this estimation, the attenuation coefficient of the difference frequency wave was ignored, because it was very small compared to those of the primary waves. Since the resonance frequencies of the main gas bubbles were supposed to be much higher than the primary frequencies, the gas void fraction in gassy sand sediment could simply be estimated by using Eq. (4) and the estimated nonlinearity parameter. Then, the gas void fraction was estimated as $\beta = 7.98 \times 10^{-4}$. This is consistent with that estimated from the sound speed variation method in gassy sand sediment.

Figure 3. Frequency spectra of (a) the signal received at a distance of 900 mm from the transmitter without gassy sand sediment and of (b) the reflected signal from gassy sand sediment in water.

CONCLUSION

The difference frequency acoustic wave in the gassy sand sediment layer was observed due to the nonlinearity of the gas bubbles. The nonlinearity parameter of the sediment layer was estimated by using the theory of the parametric acoustic array. It was $2.54 \times 10^3$, which corresponded to the gas void fraction of $7.98 \times 10^{-4}$. This value well agreed with that estimated by a linear method which was related with the sound speed variation. If the gassy sediment has a very low gas bubble concentration, the sound speed of the sediment will be almost the same with that of the water-saturated sediment. However, the nonlinear response can be dominantly observed in the gassy sediment. In this case, the nonlinear method with the difference frequency wave seems very feasible to estimate the gas void fraction in the gassy sediment.

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