NUMERICAL APPLICATION OF NONUNIFORM PSEUDO-SPECTRAL TIME-DOMAIN (PSTD) METHOD FOR ACOUSTIC SCATTERING ON RANDOMLY ROUGH OCEAN SURFACE

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ABSTRACT
The conventional pseudo-spectral time-domain (PSTD) method, using the Fourier transform which evaluates the spatial derivative of the wave equation, is an efficient technique by reason that its spatial sampling rate needs only 2 grids per wavelength. The uniform PSTD, however, is required to have more grids in case of irregular boundary in order to satisfy the numerical stability conditions. If nonuniform spatial gridding can be applied for this case, the limitation can be overcome and the numerical calculation should be more efficient. Novel nonuniform PSTD model [Liu et al, Microwave Opt Tech Lett 48, No 12, 2367-2372 (2006)] which evaluates the EM wave propagation simply by transforming the nonuniform space into uniform space using grid interpolation has been developed. In this paper, this method is used to solve the acoustic wave equation and is applied to the scattering on the rough ocean surface. The numerical results in uniform and nonuniform spatial domain are compared in terms of their accuracy and efficiency.

I. INTRODUCTION
The PSTD method which solves spatial derivatives using the Fourier transform in time domain was introduced by Kosloff and Baysal in geophysics [1] and provides exact solution for a full wave equation. While the FDTD method typically requires 10~20 grids per wavelength, the PSTD requires only 2 grids per wavelength. However, it is generally known that the Fourier PSTD method reduces its efficiency and accuracy, if the irregular boundary exists. Due to the irregular boundary, finer spatial grids are required in entire domain and the more Fourier transforms must be implemented.

As an alternative to overcome the limitation, the nonuniform PSTD should be recommended. If nonuniform spatial gridding can be applied to the Fourier PSTD method, the gridding around irregular boundaries can be adjusted to be fine and we are able to have more accurate and efficient results of the acoustic propagation and scattering. Dutt et al [2] and Liu [3] proposed the algorithm of the nonuniform FFT (Fast Fourier Transform). Leung et al, however, has found it is unstable, except that grids are nearly uniform, when they apply it to the PSTD method in electromagnetic field [4]. Novel nonuniform PSTD model which evaluates EM wave propagation simply by transforming the nonuniform space into uniform space using interpolation methods has been developed [4, 5, 6]. It is called transformed-space nonuniform PSTD (TSNU-PSTD) method. The nonuniform spatial derivative can be easily described as uniform spatial derivative using chain rule and then the relation between the two derivatives can be obtained by the interpolation method, such as Lagrange polynomial or cubic spline interpolation.

In this paper, we focus on acoustic scattering on the rough ocean surface using the TSNU-PSTD method with Lagrange polynomial interpolation. To generate randomly rough ocean surface the Pierson-Moskowitz surface roughness spectrum is used. The nonuniform grid is
applied to depth direction, while range direction should be uniformly gridded. The patterns of incident signal and scattered signal are shown. The evaluation of the scattered signal is performed by the Monte-Carlo method. We also provide comparative results of our nonuniform PSTD model and other scattering models.

II. UNIFORM AND NONUNIFORM FOURIER PSTD METHOD

The 2-D wave equation in time domain is described as

$$\rho \nabla \left( \frac{1}{\rho} \nabla p \right) = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + s(t)\delta(x)\delta(z)$$  \hspace{1cm} (Eq. 1)

where \( p(x,z,t) \) is sound pressure, \( \rho(x,z) \) is density, \( c(x,z) \) is sound speed, and \( s(t) \) is source function. In the conventional uniform PSTD method, the spatial derivative is evaluated by performing successively the forward Fourier transform and multiplying the wavenumber, followed by the inverse transform of the acoustic pressure field. That is,

$$\frac{\partial p(x)}{\partial x} = FT^{-1}[ik_xFT\{p(x)\}]$$  \hspace{1cm} (Eq. 2)

and

$$\frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) = FT^{-1} \left[ -\frac{k_x^2}{\rho} FT\{p(x)\} \right]$$  \hspace{1cm} (Eq. 3)

where \( FT \) is the forward Fourier transform and \( FT^{-1} \) is the inverse Fourier transform. For temporal derivative in (Eq. 1), we use 2nd order finite central difference method as following:

$$\frac{\partial^2 p}{\partial t^2} = \frac{p^{j+1} - 2p^j + p^{j-1}}{\Delta t^2}.$$  \hspace{1cm} (Eq. 4)

Then, we rearrange (Eq. 1), which becomes

$$p^{j+1} = \rho c^2 \Delta t^2 \left\{ \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial p^j}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial p^j}{\partial z} \right) \right\} + 2p^j - p^{j-1} - c^2 \Delta t^2 s(t)\delta(x)\delta(z).$$  \hspace{1cm} (Eq. 5)

In the TSNU-PSTD method [6], the nonuniform spatial domain \( \{x_k\} \) is transformed to the uniform domain \( \{u_k\} \) whereby the Eq. 2 should be modified as

$$\frac{\partial p(x)}{\partial x} \bigg|_{x=x_k} = \frac{\partial u}{\partial x} \bigg|_{x=x_k} = \frac{du}{dx} \bigg|_{x=x_k} FT^{-1} [ik_xFT\{p\}]_{u=u_k}.$$  \hspace{1cm} (Eq. 6)

The space transformation factor [5], \( du/dx \), in (Eq. 6) is able to be obtained by the Lagrange polynomial or cubic spline interpolation method and in this paper we use the Lagrange polynomial interpolation method. Accordingly (Eq. 5) is modified as

$$p^{j+1} = \rho c^2 \Delta t^2 \left\{ \frac{\partial u}{\partial x} \bigg|_{x=x_k} F^{-1} \left[ ik_x F \left( \frac{\partial u}{\partial x} \bigg|_{x=x_k} \right) \right] \right\} + \frac{\partial w}{\partial z} \bigg|_{z=z_k} F^{-1} \left[ ik_w F \left( \frac{\partial w}{\partial z} \bigg|_{z=z_k} \right) \right] + 2p^j - p^{j-1} - c^2 \Delta t^2 s(t)\delta(x)\delta(z).$$  \hspace{1cm} (Eq. 7)

The stability condition in two dimension can be derived when the density in (Eq. 1) is homogeneous [1], that is,

$$\Delta t \leq \frac{\sqrt{2}}{\pi} \min(\Delta x, \Delta z) \frac{c_{\text{max}}}{c_{\text{max}}}.$$  \hspace{1cm} (Eq. 8)

In case the density is inhomogeneous, it is difficult to find analytic stability condition, \( \Delta t \) should be decided empirically and be less than the value of (Eq. 8).

III. NUMERICAL EXPERIMENT FOR RANDOMLY ROUGH OCEAN SURFACE

The numerical experiment is achieved in half space region. The rough ocean surface is
generated with Pierson-Moskowitz surface roughness spectrum [7], which is represented as

\[ S(\omega) = \frac{a^2 g^2}{2\omega^2} \exp \left[ -\beta \left( \frac{g}{U_w \omega} \right)^4 \right], \quad (\text{Eq. 9}) \]

where \( a=8.10 \times 10^{-3}, \beta=0.74, \ g=9.81 \text{m/s}^2, \ w \) is the temporal frequency of the surface waves, and \( U_w \) is wind speed. Then we can have the wave height function in terms of time and space, transforming the spectrum in frequency domain to temporal and spatial domain as following:

\[ \zeta(x,t) = \sum_{n=1}^{\infty} A_n \cos(\omega_n t - k_n x + \varepsilon_n), \quad (\text{Eq. 10}) \]

where \( A_n = \sqrt{S(\omega)} \cdot d\omega \), and the phase \( \varepsilon_n \) is randomly generated. The spatial domain along depth is nonuniformly gridded. The grids around the surface should be very fine. In addition, the boundary with high contrast impedance should be smoothed.

The absorbing layer is represented by

\[ f_j = e^{-a^2 (b-j)^2}, \quad (\text{Eq. 11}) \]

where \( a \) is a coefficient of filtering, \( b \) is the number of spatial grids of the layer, and \( j \) represents grid points [8]. It is known that the optimal value of \( a \) when we set \( b \) as 40 is 0.015. These values are used for our numerical experiments.

In the conference, numerical examples on the rough boundary and comparative study with other scattering models will be presented.

IV. CONCLUSIONS

To numerically solve the scattering problem at the randomly rough ocean surface, the Fourier PSTD method in nonuniform spatial domain has been applied. This method is expected to provide the better results than the conventional uniform Fourier PSTD method in terms of accuracy and efficiency.