SPATIAL COHERENCE BETWEEN MICROPHONES WITH ARBITRARY FIRST-ORDER DIRECTIVITY IN REVERBERANT ACOUSTIC FIELDS

PACS: 43.55.Cs

Kuster, Martin; van Walstijn, Maarten
SARC, Queen’s University Belfast, BT7 1NN Belfast, Northern Ireland; m.kuster@qub.ac.uk

ABSTRACT

The theoretical expressions for the spatial correlation and spatial coherence between signals representing the pressure and the components of the particle velocity vector in a reverberant acoustic field are established in the literature and have also been corroborated with measurements [F. Jacobsen, J. Acoust. Soc. Am., 108(1), 2000]. In the present paper, expressions are derived from theory for the spatial coherence between microphone signals whose directivities can be written as a combination of pressure and particle velocity components. The resulting expressions are a combination of the already established expressions for the coherence functions between pressure and the components of the particle velocity vector. The theory has been verified on the reverberant tails of measured room impulse responses in rooms of various size and acoustic characteristic.

1 INTRODUCTION

The spatial correlation or coherence between microphone signals recorded in an enclosed space is relevant to audio engineering, where it is often referred to as cross-talk between microphone signals. A number of authors have considered the correlation between microphones in a surround sound recording setup [1, 2]. In general, theoretical expressions cannot be obtained because the correlation depends on the sound field in which the recording was performed.

If the sound field in the room is assumed to be homogenous and isotropic, it is possible to derive closed-form solutions for the spatial correlation and coherence functions. In 1954, Cook et al. [3] have presented the spatial cross-correlation between the sound pressure measured at two points in a diffuse sound field. More recently, Jacobsen [4] and Jacobsen and Roisin [5] have extended the treatment to particle velocity components and also introduced the coherence functions as a more effective measure. The spatial coherence between signals $x(r, t)$ and $y(r, t)$ is defined here as [6]

$$\gamma_{xy}^2(r, \omega) = \frac{|S_{xy}(r, \omega)|^2}{S_{xx}(0, \omega)S_{yy}(0, \omega)},$$

(1)

where $S_{xx}$ and $S_{yy}$ are power-spectral densities and $S_{xy}$ is the cross-spectral density.

Jacobsen and Roisin arrived at the following spatial coherence functions between pressure $p$ and particle velocity components parallel $v_{\parallel}$ and perpendicular $v_{\perp}$ to the vector joining the two measurement locations

$$\gamma_{pp}^2(r, \omega) = \left[ \frac{\sin(kr)}{kr} \right]^2,$$

(2a)

$$\gamma_{p_{\parallel}p_{\parallel}}^2(r, \omega) = 3 \left[ \frac{\sin(kr) - (kr)\cos(kr)}{(kr)^2} \right]^2,$$

(2b)

$$\gamma_{v_{\parallel}v_{\parallel}}^2(r, \omega) = 9 \left[ \frac{(kr)^2 \sin(kr) + 2kr \cos(kr) - 2 \sin(kr)}{(kr)^3} \right]^2.$$

(2c)
\[ 2 \begin{align*}
\gamma_{\perp,\perp}^2 (r, \omega) &= \sin (kr) - (kr) \cos (kr) \over (kr)^3. 
\end{align*} \]

In the above expressions, \( \omega \) is the angular frequency, \( k \) is the acoustic wave number and \( r \) is the length of the vector joining the two measurement locations. Note that in the last expression, the two perpendicular velocity components must be parallel to each other. The correlation and coherence for any other combination of pressure or components of the particle velocity is zero.

Jacobsen and Roisin have further validated these expressions experimentally in a reverberation chamber and noted that agreement can only be observed if in the experiments either spectral or spatial averaging is employed. The latter requirement stems from the fact that the sound field in a reverberation chamber is reverberant but not diffuse in its strict definition. Since the coherence is a measure of the linear time-invariant relationship, the sound field due to a single sound source in an enclosed space (such as the reverberation chamber) is actually fully coherent if no averaging is employed.

2 DERIVATION

The established coherence functions are now extended and coherence functions between two microphones with arbitrary first-order directivity are derived. This means that the microphone directivity can be expressed as a linear combination of pressure and particle velocity components. The derivation is performed for the 2-D case in the horizontal plane but can analogously be extended to the general 3-D case.

The two microphone signals can be written as

\[ M_x (r_x, t) = b_{w,x} W_r (r_x, t) + b_{xy,x} \cos (\theta_{M_x}) X (r_x, t) + b_{xy,x} \sin (\theta_{M_x}) Y (r_x, t), \]

\[ M_{\psi} (r_\psi, t) = b_{w,\psi} W_r (r_\psi, t) + b_{xy,\psi} \cos (\theta_{M_\psi}) X (r_\psi, t) + b_{xy,\psi} \sin (\theta_{M_\psi}) Y (r_\psi, t), \]

where the \( b \)-coefficients define the directivity and \( W, X, \) and \( Y \) (together with \( Z \)) are the B-format signals as used in Ambisonics [7] and as measured by e.g. the SoundField microphone. The directivities of the B-format signals are given by

\[ W = 1, \quad X = \sin \phi \cos \theta, \quad Y = \sin \phi \sin \theta. \]

Since the derivation is performed in the horizontal plane, \( \phi = \pi/2 \).

2.1 Decomposition into pressure and the two velocity components

In order to derive the coherence function between \( M_x \) and \( M_\psi \), it is necessary to decompose the microphone signals into terms proportional to the pressure and particle velocity components parallel and perpendicular to the vector \( r_{x,\psi} \) joining the two microphone positions as shown in Figure 1. From the figure, it can be seen that the main lobe of microphone \( M_x \) in the coordinate system \( (i_\perp, i_\parallel) \) defined by the vector \( r_{x,\psi} \) is rotated not by \( \theta_{M_x} \) but by \( \theta_{M_x} - \theta_{X,\psi} \). Note that the
directions \((i_1, i_2)\) and the value of \(\theta_{x\psi}\) vary with the relative position and direction of the two microphones.

Following Figure 2 it can then be inferred that

\[
\begin{align*}
 b_{p,x} &= b_{w,x}, \\
 b_{v_{1,x}} \cos(\theta_{M_x} - \theta_{x\psi}) &= b_{x_{y,x}} \cos(\theta_{M_x}), \\
 b_{v_{1,x}} \sin(\theta_{M_x} - \theta_{x\psi}) &= b_{x_{y,x}} \sin(\theta_{M_x}).
\end{align*}
\]

Using these relationships, the signals in Eq. (3) can be rewritten as

\[
\begin{align*}
 M_x(0, t) &= b_{p,x} \psi(0, t) + b_{v_{1,x}} \cos(\theta_{M_x} - \theta_{x\psi}) \chi(0, t) + b_{v_{1,x}} \sin(\theta_{M_x} - \theta_{x\psi}) \chi'(0, t), \tag{5a} \\
 M_{\psi}(r, t) &= b_{p,\psi} \psi(r, t) + b_{v_{1,\psi}} \cos(\theta_{M_{\psi}} - \theta_{x\psi}) \chi(r, t) + b_{v_{1,\psi}} \sin(\theta_{M_{\psi}} - \theta_{x\psi}) \chi'(r, t). \tag{5b}
\end{align*}
\]

Since only the distance between the two microphones is relevant in the ensuing derivation, \(r_x\) and \(r_{\psi}\) in these two equations have been replaced for notational convenience by the distance \(r\), which is the length of the vector \(r_{x\psi}\).

Also for notational convenience, the sine and cosine factors are incorporated into the \(b\)-coefficients by the following substitutions

\[
\begin{align*}
 b_{p,x} &\to b_{p}, \\
 b_{v_{1,x}} \cos(\theta_{M_x} - \theta_{x\psi}) &\to b_{v_{1}}, \\
 b_{v_{1,x}} \sin(\theta_{M_x} - \theta_{x\psi}) &\to b_{v_{1}}.
\end{align*}
\]

Using the relationship between the B-format and the pressure and components of the particle velocity, the microphone signals, written in terms of pressure and particle velocity components parallel and perpendicular to the vector \(r_{x\psi}\), are then finally given by

\[
\begin{align*}
 M_x(0, t) &= b_{p} p(0, t) + \rho_0 c b_{v_{1,x}} v_{\parallel}(0, t) + \rho_0 c b_{v_{1,x}} v_{\perp}(0, t), \tag{6a} \\
 M_{\psi}(r, t) &= b_{p,\psi} p(r, t) + \rho_0 c b_{v_{1,\psi}} v_{\parallel}(r, t) + \rho_0 c b_{v_{1,\psi}} v_{\perp}(r, t). \tag{6b}
\end{align*}
\]

### 2.2 Derivation of coherence function

The spatial coherence \(\gamma^2_{M_x M_{\psi}}(r, \omega)\) is derived by first calculating the auto- and cross-correlation functions. The Fourier transform of these are the power- and cross-spectral densities that form the coherence according to Eq. (1). A full derivation is beyond the scope of the current paper and is to be presented in a forthcoming paper.

The cross-correlation function \(R_{M_x M_{\psi}}(r, \tau)\) can be calculated through the expected value operator. From the linearity of the expected value operator, it follows that

\[
R_{M_x M_{\psi}}(r, \tau) = \rho_0 c b_{v_{1,x}} b_{p,\psi} R_{E_{v_{1}}} \delta(\tau) + \rho_0 c b_{v_{1,x}} b_{E_{v_{1}}} R_{E_{v_{1}}} (0, \tau) + (\rho_0 c)^2 b_{v_{1,x}} b_{v_{1,\psi}} R_{v_{1}} (r, \tau) + b_{p,x} b_{p,\psi} R_{pp} (r, \tau) + \rho_0 c b_{v_{1,x}} b_{v_{1,x}} R_{E_{v_{1}}} (0, \tau) + \rho_0 c b_{v_{1,x}} b_{v_{1,\psi}} R_{E_{v_{1}}} (r, \tau) + b_{v_{1,x}} b_{p,\psi} R_{pp} (r, \tau) + \rho_0 c b_{p,x} b_{v_{1,\psi}} R_{p v_{1}} (r, \tau) + \rho_0 c b_{p,x} b_{v_{1,\psi}} R_{p v_{1}} (r, \tau). \tag{7}
\]
By inference from Eq. (2) and the brief discussion below that equation, it follows that the terms involving $R_{v_1,v_1}(r,\tau)$, $R_{v_1,v_2}(r,\tau)$ and $R_{pp,v_1}(r,\tau)$ are zero. Further, it can be shown that $R_{pp,v_1}(r,\tau) = R_{v_1,p}(r,\tau)$ and therefore

$$R_{M_x,M_y}(r,\tau) = (\rho_0 c)^2 b_{v_1,\psi} R_{v_1,v_1}(r,\tau) + (\rho_0 c)^2 b_{v_1,\psi} R_{v_1,v_1}(r,\tau) + \rho_0 c (b_{p,x} b_{v_1,\psi} + b_{v_1,\psi} b_{p,x}) R_{p,v_1}(r,\tau) + b_{p,x} b_{p,x} R_{p,p}(r,\tau).$$

(8)

The calculation of the cross-spectral density $S_{M_x,M_y}(r,\omega)$ is not too difficult but care must be taken in the Fourier transform because $R_{pp,v_1}(r,\tau)$ has a $\sin(\omega \tau)$ factor whilst the other cross-correlation functions have a $\cos(\omega \tau)$ factor (see [4]). Using the Fourier transform of a sine and cosine function, this means that the cross-spectral density $S_{M_x,M_y}(\omega)$ has both a real and imaginary term. After some manipulation, the coherence $\gamma_{M_x,M_y}^2(r,\omega)$ then follows in terms of the equations in Eq. (2) as

$$\gamma_{M_x,M_y}^2(r,\omega) = \frac{\left| b_{p,x} b_{p,x} \gamma_{pp}(r,\omega) + b_{v_1,\psi} b_{v_1,\psi} \gamma_{v_1,v_1}(r,\omega) / 3 + b_{v_1,\psi} b_{v_1,\psi} b_{v_1,\psi} b_{v_1,\psi} \gamma_{v_1,v_1}(r,\omega) / 3 \right|^2}{\left( b_{p,x}^2 + b_{v_1,\psi}^2 / 3 + b_{v_1,\psi}^2 / 3 \right) \left( b_{p,x}^2 + b_{v_1,\psi}^2 / 3 + b_{v_1,\psi}^2 / 3 \right)} + \frac{\left| (b_{p,x} b_{v_1,\psi} + b_{v_1,\psi} b_{p,x}) \gamma_{p,v_1}(r,\omega) / \sqrt{3} \right|^2}{\left( b_{p,x}^2 + b_{v_1,\psi}^2 / 3 + b_{v_1,\psi}^2 / 3 \right) \left( b_{p,x}^2 + b_{v_1,\psi}^2 / 3 + b_{v_1,\psi}^2 / 3 \right)}.$$  

(9)

The expressions in the denominator stem from the auto-correlation functions and are essentially normalisation factors. By inserting the respective values for the $b$-coefficients, the equations listed in Eq. (2) can be recovered from Eq. (9).

3 APPLICATION TO SURROUND SOUND RECORDING SETUP

As an application of the derived coherence functions, consider the coherence between the microphones used for the reproduction of surround sound through a standard ITU-R BS.775-1 five-channel loudspeaker setup.

Room impulse responses have been measured with a SoundField MKV microphone from which the required microphone directivity is synthesised. The spectral densities needed for the measured coherence function are calculated from the room impulse responses using Welch's periodogram method. Only the part of the room impulse response where amplitudes are exponentially decaying has been used and this time limit has been found by observing the time varying statistics of the room impulse response. Due to space constraints, results are only shown for one concert hall but similar results have been obtained in a variety of measured rooms if the time limit is adjusted accordingly.

3.1 Spaced microphone array

A microphone setup with inter-microphone distances in the order of decimetres is considered. A variety of such setups are suggested by Williams [8, 9]. The position, direction and directivity of each microphone are given on the left side of Table 1. Note that all the microphones have cardioid directivity. Figure 3 shows the theoretical and measured coherence $\gamma_{xx}^2(r,\omega)$, $\gamma_{xy}^2(r,\omega)$, $\gamma_{yy}^2(r,\omega)$ and $\gamma_{xx}^2(r,\omega)$ where the subscripts denote the microphone pairs between which the coherence is

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$r_x (m)$</th>
<th>$b_{v_x}$</th>
<th>$b_{v_y}$</th>
<th>$\theta_{M_x}$ (°)</th>
<th>$\chi$</th>
<th>$r_x (m)$</th>
<th>$b_{v_x}$</th>
<th>$b_{v_y}$</th>
<th>$\theta_{M_x}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.0,44)</td>
<td>0.5</td>
<td>0.5</td>
<td>70</td>
<td>1</td>
<td>(0,0)</td>
<td>0.10</td>
<td>0.32</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>(0.0,44)</td>
<td>0.5</td>
<td>0.5</td>
<td>290</td>
<td>2</td>
<td>(0,0)</td>
<td>0.10</td>
<td>0.32</td>
<td>317</td>
</tr>
<tr>
<td>3</td>
<td>(0.23,0)</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>3</td>
<td>(0,0)</td>
<td>0.07</td>
<td>0.31</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>(-0.23,0.28)</td>
<td>0.5</td>
<td>0.5</td>
<td>156</td>
<td>4</td>
<td>(0,0)</td>
<td>0.36</td>
<td>0.57</td>
<td>134</td>
</tr>
<tr>
<td>5</td>
<td>(-0.23,-0.28)</td>
<td>0.5</td>
<td>0.5</td>
<td>204</td>
<td>5</td>
<td>(0,0)</td>
<td>0.36</td>
<td>0.57</td>
<td>226</td>
</tr>
</tbody>
</table>
considered. This particular microphone setup is aimed for a seamless linking between adjacent microphones with the minimum amount of overlap. Therefore, the coherence is essentially zero for frequencies above 500 Hz and only reaches significant values at lower frequencies. The largest values for the coherence occur between microphones 1 and 4.

3.2 Coincident microphone array

In the case of coincident microphones, $r = 0$ and Eq. (9) can be solved analytically by applying l’Hôpital’s rule repeatedly, which results in

$$\gamma^2_{M_p M_\psi}(0, \omega) = \frac{[b_{p, X} b_{p, \psi} + b_{v_\perp, X} b_{v_\perp, \psi}/3 + b_{v_\parallel, X} b_{v_\parallel, \psi}/3]^2}{(b_{p, X}^2 + b_{v_\perp, X}^2/3 + b_{v_\parallel, X}^2/3)(b_{p, \psi}^2 + b_{v_\perp, \psi}^2/3 + b_{v_\parallel, \psi}^2/3)}.$$  (10)

Unlike for the general case of $r \neq 0$, these values are independent of frequency (unless the microphone directivities change with frequency). An intuitive explanation might be that there is no characteristic distance to be compared with the varying acoustic wavelength.

For the coincident microphone setup a least-squares decoding of the B-format into the five microphone signals, as introduced in [10, 11], is considered. The resulting direction and directivities of the microphones are listed on the right side of Table 1. Using these values in Eq. (10), the following values for the frequency-independent coherence result

$$\gamma^2_{12}(\omega) = 0.09, \quad \gamma^2_{13}(\omega) = 0.61, \quad \gamma^2_{14}(\omega) = 0.11, \quad \gamma^2_{45}(\omega) = 0.28.$$  (11)

Note that the value of the coherence between microphones 1 and 3 is substantial and suggests that there is considerable overlap between the two microphones.

Figure 4 shows the theoretical and measured coherence $\gamma^2_{12}(r, \omega)$, $\gamma^2_{13}(r, \omega)$, $\gamma^2_{14}(r, \omega)$, and $\gamma^2_{45}(r, \omega)$ where the subscript denote again the microphone pairs between which the coherence is considered. The agreement between theory and measurement is not particularly good for $\gamma^2_{12}(\omega)$ but is fairly good for all other coherence functions. Also, it appears that none of the measured coherence functions are exactly constant with frequency. Instead, the values decrease steadily and to a varying degree with frequency. A possible explanation might that the microphone capsules in the SoundField microphone are not exactly coincident but are mounted with a finite separation distance $r$ in between them.

4 CONCLUSION

Theoretical expressions for the spatial coherence functions between microphone signals whose directivities can be expressed as a combination of pressure and particle velocity components have been derived. For notational simplicity, the derivation was limited to the situation where the main lobes of the microphones and the vector joining the two microphone positions were all in the same plane but the derivation can be extended to the more general 3-D case. It has also been shown
Figure 4: Theoretical (——) and measured (—) coherence averaged over receiver 140 positions for the coincident microphone setup in a concert hall, (a) $\gamma_{12}^2(\omega)$, (b) $\gamma_{13}^2(\omega)$, (c) $\gamma_{14}^2(\omega)$, (d) $\gamma_{45}^2(\omega)$.

by way of example that the expressions are valid not only in a reverberation chamber but for essentially any room impulse response when the analysis is limited to the part of the room impulse response where the amplitudes are exponentially decaying.

As examples, the coherence between microphone signals in two surround sound recording setups have been considered. For the spaced microphone setup, the coherence exhibits only significant values below 500 Hz and decreases rapidly to zero at higher frequency. With the coincident microphones setup, the values for the coherence between some microphone signals have substantial values and are independent of frequency.

References


