ACOUSTIC BEHAVIOUR OF CIRCULAR MUFFLERS WITH SINGLE INLET AND DOUBLE OPPOSITE OUTLET

PACS: 43.50.Gf

Antebas, Antoine G.; Pedrosa, Ana M.; Denia, Francisco D.; Fuenmayor, F. Javier
Departamento de Ingeniería Mecánica y de Materiales, Universidad Politécnica de Valencia, Camino de Vera s/n, 46022 Valencia, Spain
anan1@doctor.upv.es; anpedsan@dimm.upv.es; tradegu1@mcm.upv.es; ffuenmay@mcm.upv.es

ABSTRACT

In this work, a three-dimensional analytical model for the propagation of sound in circular chamber mufflers with two opposite outlets is derived. The procedure is based on the mode matching method, which is considered for the calculation of the muffler transmission loss by matching the acoustic pressure and normal velocity across each geometrical discontinuity. The consideration of the analytical integral relations for Bessel functions and the Graf’s addition theorem allows the calculation of the acoustic performance of the muffler with low computational requirements. To validate the procedure, the analytical results are compared with finite element predictions, showing a good agreement. The acoustic attenuation performance is then examined in detail as a function of the relative location of the inlet/outlets and area ratio. Some comparisons are also given with limiting cases, such as simple expansion chambers and reversing chamber mufflers.

INTRODUCTION

The acoustic behaviour of exhaust mufflers strongly depends on some geometrical aspects such as the area ratio between ducts and their relative position [1, 2]. Although plane wave models are available for the prediction of the sound attenuation of mufflers at low frequencies, multidimensional analytical techniques [3, 4] and numerical methods [5-8] are required for higher frequencies to account for the propagation of higher order modes, the former providing a reduced computational effort.

A great amount of literature can be found regarding the multidimensional analytical modelling of mufflers. The concentric expansion chamber is considered in detail in the work of Selamet and Radavich [9], and some references can be found including the presence of duct extensions in concentric configurations [10, 11]. The effect of the offset associated with the inlet and outlet is also dealt with in the references [3, 12, 13]. Analytical models are also available for double outlet mufflers [14, 15], which exhibit a slight reduction in the acoustic performance but a better behaviour regarding flow noise and back pressure in comparison with single outlet configurations. In the previous works, the inlet and outlet(s) are located on separate endplates. For long chambers without extensions, an acoustic attenuation characterized by repetitive attenuation domes associated with reflections in the axial direction is obtained [9, 14]. The presence of extended inlet/outlet ducts produces a dome-like behaviour combined with some resonant peaks of a quarter-wave resonator [10]. This repetitive behaviour is present below the cut-off frequency of the first higher order mode. The propagation of transversal modes is known to produce a general collapse of the attenuation and therefore reduces the acoustic performance of the muffler. To obtain a partial solution to this problem, a suitable offset of the inlet/outlet ducts can be introduced in the design, which eliminates the propagation of some detrimental modes, thus providing an improved attenuation performance [13, 15].

On the other hand, reversing chambers in which both the inlet and outlet ducts are located on the same endplate, have been analytically studied in the references [4, 16, 17]. The attenuation exhibits a repetitive wide-peak behaviour for long chambers. Once the propagation of higher order modes appears, this behaviour is lost and a typical reduction in the attenuation is observed in general. A proper location of the inlet/outlet ducts can be used to extend the frequency range with wide peaks, as commented previously in the case of chambers with the inlet and outlet(s) located on separate endplates.
The objective of the present work is to investigate the acoustic performance of circular chamber mufflers with double outlet located on different endplates, thus providing a geometry which combines some features of expansion and reversing chambers. The three-dimensional analytical approach developed to carry out the prediction of the acoustic performance is presented in section 2. The validation of the procedure is shown in section 3 by comparison with finite element results. A number of geometries are then analyzed, including the effect of inlet/outlet locations as well as the area ratio between the ducts involved. In addition, comparison is also given with different chamber configurations analyzed in previous studies. To finish the work, some concluding remarks are presented in the last section.

MATHMATICAL APPROACH

Figure 1 shows the geometry of a circular chamber muffler with single inlet and double opposite outlet.

\[
\nabla^2 p + k_0^2 p = 0 \quad \text{(Eq. 1)}
\]

with \( p \) is the acoustic pressure, \( \nabla^2 \) is the Laplacian operator and \( k_0 \) is the wavenumber, defined as the ratio of the angular frequency \( \omega \) and the speed of sound \( c_0 \). The solution of Eq. (1) may be expressed, for a duct \( i \) (with \( i = 1, 2, 3, 4 \)), as the combination of a wave \( A_i \) travelling in the positive \( z \)-direction and a wave \( B_i \) travelling in the negative \( z \)-direction, yielding

\[
P_i(r, \phi, z) = P_{A_i} + P_{B_i} \quad \text{(Eq. 2)}
\]

where \( P_{A_i} \) and \( P_{B_i} \) are given by [14]

\[
P_{A_i} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{i,m,n} e^{-im\phi} J_m \left( \frac{\alpha_{mn} r_i}{R_i} \right) e^{-j\beta_{mn} z} + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} A_{-i,m,n} e^{im\phi} J_m \left( \frac{\alpha_{mn} r_i}{R_i} \right) e^{j\beta_{mn} z} \quad \text{(Eq. 3)}
\]

\[
P_{B_i} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{i,m,n} e^{-im\phi} J_m \left( \frac{\alpha_{mn} r_i}{R_i} \right) e^{j\beta_{mn} z} + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} B_{-i,m,n} e^{im\phi} J_m \left( \frac{\alpha_{mn} r_i}{R_i} \right) e^{-j\beta_{mn} z} \quad \text{(Eq. 4)}
\]

In Eqs. (2)-(4), \((r, \phi, z)\) are cylindrical co-ordinates, \( m \) and \( n \) are the azimuthal and radial mode numbers, \( A_{i,m,n} \), \( A_{-i,m,n} \), \( B_{i,m,n} \) and \( B_{-i,m,n} \) are complex modal coefficients, \( J_m \left( \alpha_{mn} r_i / R_i \right) \) is the Bessel function of the first kind and order \( m \) and \( \alpha_{mn} \) is the \( n \)-th root satisfying the rigid wall boundary condition given by \( \partial J_m \left( \alpha_{mn} r_i / R_i \right) / \partial r \bigg|_{r=R_i} \). The axial wavenumber \( k_{i,m,n} \) is given by

\[
k_{i,m,n} = \begin{cases} \sqrt{k_0^2 - (\alpha_{mn}/R_i)^2} \quad & k_0 \geq \alpha_{mn}/R_i \\ -\sqrt{k_0^2 - (\alpha_{mn}/R_i)^2} \quad & k_0 \leq \alpha_{mn}/R_i \end{cases} \quad \text{(Eq. 5)}
\]

with the sign change in Eq. (5) providing an exponential decay of the waves in \( z \).

The axial velocity of the waves \( A_i \) and \( B_i \) is obtained from the linearized Euler equation [1],

\[
j \rho_0 c_0 U = -\partial p / \partial z \quad \text{yielding}
\]

\[
U_i(r, \phi, z) = U_{A_i} + U_{B_i} \quad \text{(Eq. 6)}
\]

with

\[
\rho_0 \frac{\partial p}{\partial t} = \rho_0 c_0^2 \frac{\nabla^2 p}{\nabla^2} \quad \text{(Eq. 7)}
\]

\[
\nabla \cdot \mathbf{U} = 0 \quad \text{(Eq. 8)}
\]

\[
\nabla^2 \mathbf{U} = -\nabla p \quad \text{(Eq. 9)}
\]

\[
\mathbf{U} = \sum_{i=1}^{4} U_i(r, \phi, z) \quad \text{(Eq. 10)}
\]

\[
\mathbf{p} = \sum_{i=1}^{4} p_i(r, \phi, z) \quad \text{(Eq. 11)}
\]
\[
U_A = \frac{1}{\rho_0 \rho} \left( \sum_{m=0}^{\infty} k_{1,mm} A_{1,mm}^+ e^{-j\omega \rho \alpha_{1,mm}} + \sum_{m=0}^{\infty} k_{1,mm} A_{1,mm}^- e^{j\omega \rho \alpha_{1,mm}} \right) \left( \frac{\alpha_{1,mm} R_1}{R_i} \right) e^{-j\omega \rho \alpha_{1,mm}} \] (Eq. 7)

\[
U_B = \frac{1}{\rho_0 \rho} \left( \sum_{m=0}^{\infty} k_{1,mm} B_{1,mm}^+ e^{-j\omega \rho \alpha_{1,mm}} + \sum_{m=0}^{\infty} k_{1,mm} B_{1,mm}^- e^{j\omega \rho \alpha_{1,mm}} \right) \left( \frac{\alpha_{1,mm} R_1}{R_i} \right) e^{-j\omega \rho \alpha_{1,mm}} \] (Eq. 8)

\[\alpha_0\] being the density of the air.

In order to evaluate the complex modal coefficients \(A_{1,mm}^+, A_{1,mm}^-, B_{1,mm}^+, B_{1,mm}^-\) associated with the four ducts involved in the problem under analysis, the conditions satisfied by the acoustic fields are taken into account. Applying continuity of the acoustic pressure in the left endplate, gives

\[
P_2 \bigg|_{z=0} = P_1 \bigg|_{z=0} \text{ on } S_1 \quad \text{and} \quad P_2 \bigg|_{z=0} = P_3 \bigg|_{z=0} \text{ on } S_3 \] (Eqs. 9, 10)

Similarly, the conditions for the axial velocity in the left endplate may be written as

\[
U_2 \bigg|_{z=0} = U_1 \bigg|_{z=0} \text{ on } S_1, \quad U_2 \bigg|_{z=0} = U_3 \bigg|_{z=0} \text{ on } S_3 \quad \text{and} \quad U_2 \bigg|_{z=0} = 0 \text{ on } S_2 - S_1 - S_3 \] (Eqs. 11-13)

For the right endplate the conditions yield, for the pressure field,

\[
P_2 \bigg|_{z=L_2} = P_4 \bigg|_{z=L_2} \text{ on } S_4 \] (Eq. 14)

and for the axial velocity

\[
U_2 \bigg|_{z=L_2} = U_4 \bigg|_{z=L_2} \text{ on } S_4 \quad \text{and} \quad U_2 \bigg|_{z=L_2} = 0 \text{ on } S_2 - S_4 \] (Eqs. 15, 16)

The application of the mode matching method is now described. For each surface \(S_i\) where the continuity condition of the acoustic pressure can be applied, this condition is multiplied by the weighting function \(J_i(\alpha_{0,r}/R_1) e^{j\omega \rho s}\), for \(t = 0, 1, 2, \ldots\) and \(s = 0, 1, 2, \ldots\), and integrated over \(S_i\), yielding (for example in \(S_1\))

\[
\int_{S_1} P_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_1) e^{j\omega \rho s} dS = \int_{S_1} P_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_1) e^{j\omega \rho s} dS \] (Eq. 17)

The same condition is multiplied by \(J_i(\alpha_{0,r}/R_1) e^{-j\omega \rho s}\), for \(t = 1, 2, \ldots\) and \(s = 0, 1, 2, \ldots\), and integrated over \(S_i\), giving (for \(S_1\))

\[
\int_{S_1} P_2 \bigg|_{z=0} J_i(\alpha_{0,r}/R_1) e^{j\omega \rho s} dS = \int_{S_1} P_2 \bigg|_{z=0} J_i(\alpha_{0,r}/R_1) e^{j\omega \rho s} dS \] (Eq. 18)

The procedure is repeated for the continuity condition of the acoustic pressure at the outlet located in the left endplate \(S_3\), Eq. (10), and the outlet located in the right endplate \(S_4\), expressed by Eq. (14). The application of the mode matching technique to the velocity conditions in the left endplate, expressed in Eqs. (11)-(13), is carried out multiplying these equations by a first weighting function given by \(J_i(\alpha_{0,r}/R_2) e^{j\omega \rho s}\), for \(t = 0, 1, 2, \ldots\) and \(s = 0, 1, 2, \ldots\). The integrals are carried out over \(S_1\) for Eq. (11), \(S_3\) for Eq. (12) and \(S_2 - S_1 - S_3\) for Eq. (13). After adding the integrations, yields

\[
\int_{S_1} U_2 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{j\omega \rho s} dS = \int_{S_1} U_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{j\omega \rho s} dS + \int_{S_3} U_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{j\omega \rho s} dS \] (Eq. 19)

The consideration of the second weighting function \(J_i(\alpha_{0,r}/R_2) e^{-j\omega \rho s}\), for \(t = 1, 2, \ldots\) and \(s = 0, 1, 2, \ldots\), leads to

\[
\int_{S_1} U_2 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{-j\omega \rho s} dS = \int_{S_1} U_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{-j\omega \rho s} dS + \int_{S_3} U_1 \bigg|_{z=0} J_i(\alpha_{0,r}/R_2) e^{-j\omega \rho s} dS \] (Eq. 20)

The same procedure is applied to the velocity conditions in the right endplate expressed by Eqs. (15) and (16).
The computational requirements of the procedure are strongly reduced for mufflers involving circular ducts since it is possible to perform each integration analytically, considering the analytical integral relations for Bessel functions in combination with the Graf’s addition theorem [18]. The algebraic system associated with the previous set of integral equations may now be solved. First, each modal expansion for \( P_i \) and \( U_i \) must be truncated appropriately, with the highest mode numbers given by \( m_{\text{max}} \) and \( n_{\text{max}} \). To obtain the same number of equations and unknowns, the values \( t_{\max} = m_{\text{max}} \) and \( s_{\max} = n_{\text{max}} \) are considered for the weighting functions. After setting an incident plane wave given by \( A_{i,00}^{\pm} = 1 \) and \( A_{i,0m}^{\pm} = 0 \) for the rest of the values of \( m \) and \( n \), and two anechoic terminations defined by \( A_{j,4m}^{\pm} = \pm \delta_{m} A_{j,4m}^{\pm} \) for all \( m \) and \( n \), a system of \( 5 \left(2m_{\text{max}} + 1\right) \left(2n_{\text{max}} + 1\right) \) equations and unknowns is obtained. The transmission loss of the muffler is given by [14]

\[
TL = -10 \log \left( \frac{R_1}{R_4} \right)^2 \sum_{m=0}^{n_{\text{max}}} B_{1,0m}^{\pm} e^{\mu_{j,m}(-L_4)} \left[ \left( R_2 \right)^2 \sum_{n=0}^{m_{\text{max}}} A_{2,0n}^{\pm} e^{-\mu_{j,n}(-L_2)} \right]^2
\]  
(Eq. 21)

RESULTS AND DISCUSSION
To validate the analytical approach, a reference geometry has been considered with \( R_1 = R_3 = R_4 = 0.02 \) m, \( R_2 = 0.091875 \) m, \( L_1 = L_3 = L_4 = 0.1 \) m, \( L_2 = 0.3 \) m, \( \delta_1 = \delta_3 = \delta_4 = 0.045 \) m, \( \varphi_0 = 180^\circ \); and \( \varphi_0 = \varphi_4 = 0^\circ \). The validation of the analytical procedure is presented in Figure 2. Analytical results and FEM calculations show an excellent agreement, thus giving a validation for the analytical procedure.

Figure 2.- Validation of the method: \( \text{------} \), Analytical approach; \( \text{-----} \), FEM.

Position of the ducts
Figure 3 shows the attenuation of three geometries with different duct locations. The dimensions are those considered for the reference geometry of Figure 2, with some variations to study the effect of the position of the ducts. These are: geometry 1, \( \varphi_0 = 180^\circ \) and \( \varphi_0 = \varphi_4 = 0^\circ \); geometry 2, \( \varphi_0 = 90^\circ \), and \( \varphi_0 = \varphi_4 = 0^\circ \); and geometry 3, \( \varphi_0 = 90^\circ \), \( \varphi_0 = 0^\circ \), and \( \varphi_4 = 180^\circ \). Fig. 3 illustrates that the worst acoustic attenuation appears for geometry 1, due to the propagation of the (1,0) first asymmetric mode. The location of the inlet at \( \varphi_0 = 90^\circ \) can improve the acoustic attenuation performance and extends the dome-like behaviour up to the onset of the (2,0) asymmetric mode.

Addition of the second outlet in the opposite endplate
Six configurations are considered here with \( R_1 = R_3 = 0.0268 \) m, \( R_2 = 0.091875 \) m, \( L_1 = L_3 = L_4 = 0.1 \) m, \( L_2 = 0.3 \) m, \( \delta_1 = 0 \) m, \( \delta_3 = \delta_4 = 0.057661 \) m (optimum offset distance [14]), and \( \varphi_0 = \varphi_3 = \varphi_4 = 0^\circ \). Different radii are considered for the outlet in the opposite endplate: \( R_4 = 0 \) m (reversing chamber muffler), \( R_4 = 0.005 \) m, \( 0.01 \) m, \( 0.015 \) m, \( 0.02 \) m and \( 0.0268 \) m. Figure 4 shows a transition from a reversing chamber muffler to a simple expansion chamber behaviour, as \( R_4 \) is increased. The attenuation is then reduced when a second outlet is added in the opposite endplate. However, a reduction of the flow noise and back pressure is expected.
Figure 3.- Position of the ducts: —, geometry 1; ——, geometry 2; ——, geometry 3.

Figure 4.- Addition of the second outlet in the opposite endplate: ——, $R_4 = 0 \text{ m}$; ——, $R_4 = 0.005 \text{ m}$; ——, $R_4 = 0.01 \text{ m}$; ——, $R_4 = 0.015 \text{ m}$; ——, $R_4 = 0.02 \text{ m}$; ——, $R_4 = 0 \text{ m} = 0.0268 \text{ m}$.

Addition of the second outlet in the inlet endplate
The same analysis as in the previous section is now repeated for $R_4 = 0.0268 \text{ m}$, $R_4 = 0 \text{ m}$ (simple expansion chamber), and $R_3 = 0.005 \text{ m}, 0.01 \text{ m}, 0.015 \text{ m}, 0.02 \text{ m}$ and $0.0268 \text{ m}$. Figure 5 shows that the addition of a second outlet in the inlet endplate delivers very slight modifications in the transmission loss. Again, a reduction of the flow noise and back pressure is expected.

Effect of the section of the outlets
Four configurations are considered with $R_1 = 0.0268 \text{ m}, R_2 = 0.091875 \text{ m}, L_1 = L_3 = L_4 = 0.1 \text{ m}, L_2 = 0.3 \text{ m}, \delta_1 = 0 \text{ m}, \delta_3 = \delta_4 = 0.057661 \text{ m}$, and $\phi_{i1} = \phi_{i3} = \phi_{i4} = 0^\circ$. Different radii are considered for the outlets: $R_3 = R_4 = 0.005 \text{ m}, 0.015 \text{ m}, 0.02 \text{ m}$ and $0.0268 \text{ m}$. Fig. 6 shows that a higher attenuation is obtained in all the frequency range when lower radii are considered for the outlet ducts, as expected.
Figure 6.- Effect of the section of the outlets: ---, \( R_3 = R_4 = 0.005 \) m; ----, \( R_3 = R_4 = 0.015 \) m; ---, \( R_3 = R_4 = 0.02 \) m; ----, \( R_3 = R_4 = 0.0268 \) m.

CONCLUSIONS
A 3D analytical model for the prediction of the acoustic attenuation performance of circular chamber mufflers with two opposite outlets has been derived. The analytical model has been validated by comparison with finite element predictions, and then applied to the analysis of the acoustic performance of this configuration of muffler as a function of the relative location of the inlet/outlets and area ratio.

ACKNOWLEDGEMENTS
Financial support of Conselleria d’Empresa, Universitat i Ciència (GV/2007/133) and Vicer. de Investigación, Desarrollo e Innovación of UPV (PAID-06-06) is gratefully acknowledged.

References: