LDV MEASUREMENT OF FAST ACOUSTIC STREAMING IN A STANDING WAVE GUIDE.

PACS: 43.25.Nm

Moreau, Solenn¹; Bailliet, Hélène¹; Valière, Jean-Christophe¹
¹ Laboratoire d’Etudes Aérodynamiques; Bâtiment K, 40 avenue du Recteur Pineau, 86022 Poitiers Cedex, France; solene.moreau@lea.univ-poitiers.fr, helene.bailliet@lea.univ-poitiers.fr, jean-christophe.valiere@lea.univ-poitiers.fr

ABSTRACT
Acoustic streaming is measured in an acoustic field set in a cylindrical wave guide having an acoustic driver at each end. The axial streaming velocity is measured using Laser Doppler Velocimetry (LDV). Nonlinear effects are characterized by an appropriate Reynolds number \( Re_{NL} = \left( \frac{U}{c} \right)^2 \left( \frac{R}{\delta_v} \right)^2 \) with \( U \) the acoustic velocity amplitude, \( c \) the speed of sound, \( R \) the tube radius and \( \delta_v \) the boundary layer thickness. The case \( Re_{NL} \ll 1 \) corresponds to the so-called “slow streaming” and the case \( Re_{NL} > 1 \) is referred to as nonlinear streaming or “fast streaming”. The variation of axial streaming velocity with respect to the transverse and axial coordinate are compared to available slow streaming theories. Streaming is measured both in the core region and in the acoustic viscous boundary layer. Streaming velocity in the center of the guide agrees reasonably well with slow streaming theories for small \( Re_{NL} \) but deviates significantly from such predictions for fast streaming. Across the whole guide section, two vortices are measured for \( Re_{NL} < 70 \) as predicted by slow streaming theories. But, when the Reynolds number is increased, two new vortices appear in the near wall region.

INTRODUCTION
Acoustic streaming is a second order steady velocity that is induced by and superimposed on the dominating first order acoustic velocity.

Figure 1: Streaming velocity field for \( \frac{R}{\delta_v} \approx 4 \). In (a), the acoustic streaming vortices in a resonator is schematized. In (b), the variation of axial streaming velocity with respect to the transverse coordinate \( r \) is schematized for \( x = \frac{1}{4} \).

In a two-dimensional resonator with rigid walls in which a \( \frac{1}{4} \) standing wave is set up, streaming vortices are present either sides of the central axis with a periodicity of \( \frac{1}{4} \), as shown by Fig. 1 (a), with \( \lambda = \frac{2\pi}{f_{ac}} \) the acoustic wave length, \( c \) the celerity of sound and \( f_{ac} \) the acoustic frequency. Vortices present in the center region of the guide (for \( R - 5\delta_v < r < -R + 5\delta_v \)) correspond to the outer streaming with \( R \) the tube radius, \( \delta_v = \sqrt{2\nu/\omega} \) the boundary layer thickness, \( \nu \) the kinematic
viscosity of the fluid and $\omega = 2\pi f_{ac}$ the angular acoustic frequency. In the near wall region, the inner streaming vortices have directions of rotation opposite to those of the outer.

Streaming was first modeled by Rayleigh [5] in 1883 for wide channels (in which the boundary layer thickness is negligible in comparison to channel width and the wavelength is big compared to the tube radius). His solution describes the steady vortices outside the boundary layer, commonly referred to now as Rayleigh streaming. Thermal effects on the outer streaming were first considered by Rott [6]. Because its importance in thermoacoustic engines and refrigerators, streaming has been reconsidered theoretically during the last years without restricting to outer vortices and to wide channels by several authors. Among them, Bailliet et al. [1] included temperature dependence of the viscosity in addition to heat conduction in their analysis of inner and outer streaming in both two-dimensional channels and cylindrical tubes. In Bailliet et al. studies, the mean temperature was authorized to vary along the channel walls. For the case of a wide tube ($R > \delta_v$), their results were in agreement with the results of Rott.

All the above cited studies assume the streaming to be very slow, that is slow enough to leave the first-order variables unperturbed. As the streaming becomes larger, recent theoretical work by Menguy and Gilbert [3] indicates that non-linear effect of fluid inertia will render the vortex pattern distorted. In the case of a waveguide, this effect is determined by an appropriate Reynolds number

$$Re_{NL} = \left(\frac{U}{c}\right)^2 \left(\frac{R}{\delta_v}\right)^2.$$  

(Eq. 1)

with $U$ the acoustic velocity amplitude. $Re_{NL}$ compares inertia and viscosity and determines the degree to which the streaming velocity field is distorted relative to the field in the slow streaming case. The case $Re_{NL} < 1$ corresponds to the slow streaming and the case $Re_{NL} \geq 1$ is referred to as 'nonlinear streaming' or fast streaming.

Besides these theoretical studies, experimental works on streaming have been rather scarce and qualitative until recent development of laser techniques. For $Re_{NL} \approx 1$, the outer streaming associated with one-dimensional monofrequency acoustic standing wave in a resonator has been investigated using Particle Image Velocimetry (PIV) (by Campbell et al. [2]). Their results agreed reasonably well with Rott’s predictions. Very recent studies by Thompson et al. [7] reported experimental studies of outer Rayleigh streaming in a guide using LDV for $2 < Re_{NL} < 20$. Their results for $Re_{NL} \approx 1$ showed that streaming velocities are in better agreement with the theory of Rott than with the prediction of Menguy and Gilbert, suggesting that the influence of fluid inertia on the streaming-velocity field is not as deterministic as thermal conditions. They considered three different thermal conditions: isothermal, uncontrolled and insulated. For $Re_{NL} > 1$, they observed that when the magnitude of the thermoacoustically induced temperature gradient increases, the magnitude of the streaming decreases and the shape of the streaming cell becomes increasingly distorted.

In spite of these important recent advancements and although streaming has been reconsidered theoretically during the last years without restricting to outer vortices, there is still no experimental evidence and characterization of inner vortices in standing wave guides. In the present work, outer and inner streaming velocity field generated by an acoustic standing wave in a cylindrical resonator are investigated experimentally by means of Laser Doppler Velocimetry (LDV). Next section provides an overview of the method used for calculating streaming velocity and the last section presents results of measurements together with their comparison to the available theoretical expectation.

**PROCEDURE**

**Experimental apparatus**

---

**Figure 2: Diagram of the experimental apparatus**

---

2

19th INTERNATIONAL CONGRESS ON ACOUSTICS – ICA2007MADRID
The setup used to observe the phenomenon of acoustic streaming is shown in Fig. 2 and consists in a cylindrical (2D) tube connected at each end to a loudspeaker so that a \( \frac{1}{2} \) high level standing wave is sustained in the guide. Loudspeakers are connected to each end of the wave guide via convergents designed to avoid separation effect related to the singularities in change of section. The main part of the wave guide is a glass cylinder of inner radius \( R = 19.5 \text{mm} \) that is simply surrounded by air corresponding to an uncontrolled boundary thermal condition. The total length of the wave guide \( L = \frac{1}{2} \) is 1.9m so that the frequency \( f_{ac} \) is 88Hz. For this frequency, the boundary layer thickness \( \delta_{bc} \), of order of 0.2mm, is a very small fraction of the acoustic wavelength \( \lambda \). Because streaming outer vortices have a \( \frac{1}{2} \) periodicity, we expect two Rayleigh streaming cells along the guide length. The wave guide is filled with atmospheric air. Wood smoke is introduced into the guide to render the flow visible. A wave generator provides the loudspeaker input signal, whose frequency and amplitude are controlled, as well as a trigger reference signal, used to synchronize the LDV system. The parameters of the LDV system are adjusted for sound measurement [4]. The axial particle velocity is measured both along the centerline of the guide and across the section for several axial positions (with a 0.05mm step near the wall and with gradually growing steps until 5mm in the center of the guide).

**Determination of axial streaming velocity**

\[
\begin{align*}
\text{(a)} & \quad \text{Fig. 3 represents the method used to compute streaming velocity: Axial velocity issued from LDV measurement } u_i \text{ (Fig. 3(a)) is brought back on one acoustic period } T_{ac} \text{ (Fig. 3(b) and (d))} \\
& \text{sorted per growing time and averaged over regular time step on the acoustic period (Fig. 3(c) and (e))} \\
& \text{with } N_k \text{ the particle number for each step } k \text{ and } N \text{ the number of steps. The time step used for averaging is chosen to be the maximum time between two measurements in the acoustic period. Streaming axial velocity } u_2 \text{ is then calculated as the average of velocity points over the period} \\
& \quad u_2(r) = \frac{1}{N} \sum_{k=1}^{N} u_k(r, t_k). \\
\end{align*}
\]

Preliminary studies show that measurements should be performed more than 26 min after the acoustic field is switched on and are stable after this time. Once the steady-state has been achieved, the LDV measurements are performed.

---

**Figure 3:** LDV signal method. In (a) the velocity of rough data from LDV measurement are presented as a function of time. In (b), the axial velocity is brought back on one acoustic period. In (c), the axial velocity is averaged over regular time step on the acoustic period. (d) and (e) are zoomed views of (b) and (c).
reached, it is important to evaluate how many samples are needed to insure convergence of streaming velocity. Again, preliminary studies show that if the calculation of acoustic streaming is performed over 40000 samples or more then the calculated value of $u_2$ convergence is reached.

RESULTS

Outer streaming behavior

![Graph](image1)

Figure 4: Axial streaming velocity in guide section for $Re_{NL} = 0.4$; —: Rayleigh theory; •: measurements. In (a), the variation of axial streaming velocity with respect to the axial coordinate $x$ ($r = 0$) is compared to the theory of Rayleigh. In (b) and (c), the variation of axial streaming velocity with respect to the transverse coordinate $r$ ($x = \lambda/8$ and $3\lambda/16$) is compared to the theory.

![Graph](image2)

Figure 5: Axial streaming velocity in guide section for $Re_{NL} = 98$; —: Rayleigh theory; •: measurements. In (a), the variation of axial streaming velocity with respect to the axial coordinate $x$ ($r = 0$) is compared to the theory of Rayleigh. In (b) (c) and (d), the variation of axial streaming velocity with respect to the transverse coordinate $r$ ($x = \lambda/16$, $\lambda/8$ and $3\lambda/16$) is compared to the theory.

Fig. 4 and Fig. 5 represent the axial streaming velocity in the guide section. Theoretical expression for the streaming derived by Rayleigh is also represented for comparison. At low level (Fig. 4) the measured axial streaming velocity in the center of the guide is a bit higher than the theoretical
curves in agreement with the measurements of Thompson et al. [7]. When the acoustic level is increased, the measured axial streaming velocity in the center of the guide tends to be equal and then smaller than theoretical expectations. In our experiments, measurements were performed for higher $Re_{NL}$ than in any previous studies. For fast streaming (Fig. 5), the measured axial streaming velocity in the center of the guide tends to zero that is not in agreement with any available theory, and, in agreement with [7] its evolution is mostly driven by the thermal condition (uncontrolled in our experiments).

![Figure 6](image_url)

Figure 6: Evolution of axial component of the centerline streaming velocity ($r = 0$) dimensioned by the Rayleigh centerline axial streaming velocity $u_{2, Rayleigh,c}$ as a function of the Reynolds number of streaming motion $Re_{NL}$. This diagram shows that the radial dependence of $u_2$ departs from parabolic ($u_{2,c}/u_{2, Rayleigh,c} = 1$) as $Re_{NL}$ increases. Fig. 6 shows different changes in the slope of the curve that gives the centerline streaming velocity as a function of the $Re_{NL}$. So, different critical Reynolds number appear for the axial component of the centerline streaming velocity that depend on the axial position along the tube axis.

**Inner streaming vortices in the near wall region**

Let us turn now to the inner vortices, that are near wall streaming vortices whose directions of rotation are opposite to those of the outer cells (see Fig. 1 (a)) and that have never been measured before. When the velocity amplitude is increased, different stages for the generation and evolution of these inner streaming vortices can be determined. In order to focus on inner vortices, we zoom over the near wall region and consider the streaming velocity $u_2$ at a distance from 0 to about $30 \delta_v$ from the wall. Recall that a streaming vortex is detected in the guide when the streaming velocity crosses the abscissa axis $r = 0$, corresponding to the center of the vortex (see Fig. 1 (b)).

Fig. 7 represents the axial streaming velocity dimensioned by the Rayleigh centerline axial streaming velocity $u_{2, Rayleigh,c}$ (normalized streaming velocity) as a function of the distance to the wall $R - r$ dimensioned by the thickness of the boundary layer $\delta_v$ for $x = \lambda/16$. Theoretical expression for the streaming derived by Rayleigh and Bailliet et al. are also represented for comparison (Bailliet et al. theory is valid both in the far and in the near wall region whereas Rayleigh theory is only valid for outer streaming; both assume $Re_{NL} << 1$); the shapes of these theoretical curves are similar for the outer streaming vortices.

Fig. 7 shows that only one vortex is measured over the half section of the guide for $Re_{NL} = 29$ and $Re_{NL} = 51$ whereas two vortices are measured in the near wall region for $Re_{NL} = 98$ and $Re_{NL} = 118$. In Fig. 7, the arrow visualises the evolution of streaming velocity maximum for increasing $Re_{NL}$. When the Reynolds number is increased, the maximum normalized velocity decreases and moves towards the guide center. Therefore the outer vortex becomes narrower and its maximum normalized velocities both in the center and in the near wall region keeps on decreasing so that new vortices are created in the near wall region. The evolution is the same for $x = \lambda/24$ but was found to be a bit different for $x = \lambda/8$ and $x = 3\lambda/16$.
CONCLUSION

Axial streaming velocity was measured by LDV and its evolution with respect to both the transverse coordinate $r$ and the axial coordinate $x$ was studied for increasing Reynolds number. When the Reynolds number is increased, the velocity maxima of the outer streaming vortices in the guide center and in the near wall region decrease. For fast streaming, streaming velocity profile departs from parabolic Rayleigh theory and the centerline streaming velocity tends to zero.

In the present study, inner streaming was also observed. The variation of axial streaming velocity dimensioned by the Rayleigh centerline velocity in the near wall region with respect to the transverse coordinate $r$ was measured. Two new inner streaming vortices were observed in the near wall region for $Re_{NL} \geq 70$ and for $x = \frac{\lambda}{24}$ and $x = \frac{\lambda}{16}$.

References


