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INTRODUCTION TO THE ACOUSTIC STUDY OF THE PERUVIAN QUENA

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ABSTRACT
The quena is a very ancient Peruvian musical wind instrument, arises in the pre-Inca era created by some of the cultures that conformed the territory of ancient Peru. This musical instrument is made in empirical way with good results of sound. This work described the mathematically acoustic behavior of the principal parts of the quena, such as the blow end and the tonal holes. The mathematical model allows finding the physical length of the quena and the location of the tonal holes. The equations of the model, consider end corrections due to the tonal holes, to the open end of the duct, to the blow end and the small cavities of air that are formed when the player cover the tonal holes with his fingers.

INTRODUCTION
Peru have a rich legacy in musical ancient instruments, they were very related to the natural environment. The quena is an instrument that in its sounds encloses a halo of mystery, myth and legend, but it does not escape to the possibility of studying it from the acoustic approach. The present study is based on simple Eqs. that describe the behavior of the air columns that are formed in the principal duct, the small ducts of the tonal holes and the duct formed by the lips and the blow end. In addition, we showed that it is possible to improve the current design of the quena with a physical mathematical concept. This model suggests, the elimination of the end wood cover perforated that traditionally is placed in the open end of the tube to compensate the lack of physical length or the bad distribution of the tonal holes, which can introduce distortions in the tones for the second and third octave. Another simultaneous mistake that the manufacturers commit is to enlarge the diameter of the tonal holes to achieve the tune-up, to such point that impedes the fingering of the same ones and that the melodious sound of the quena derives in a very strong and noisy sound that goes out of the telluric and Andean concept.

LIST OF SYMBOLS

ρ	Air density
c	Sound velocity in the air
v_c	Sound velocity inside the duct
a	Radius of the duct
b	Radius of the tonal hole
d	Equivalent radius of the blow end, where $d = (1+\pi/4)^{1/2} a$
M_E	Effective length of the duct, from the open end to the center of the first tonal hole
t_d	Effective thickness of the tube due to the blow end
t_e	Effective thickness of the tube due to the open side hole
β	Fraction of blow end covered by the player's lip. In the case of the quena it is 11,20% to 22,4%
Δl_d	End correction due to the blow end
l_0	Physical length of duct measured from the top end to the open end
Δl_1	End correction due to the drill of the first tonal hole
Δl_k	End correction due to the cavity of volume V, that is formed when the tonal holes are covered
l_{E1}	Effective distance between the top end and the end correction due to the first open hole

L_0	Physical length of duct from the top end to the open end
$2s$	Spacing between two tonal holes
Δl_s	End correction due to the duct formed between the first open hole and the last closed hole

DIMENSIONAL CHARACTERIZATION OF THE QUENA

The quena consists of a tube of internal diameter usually less that 20 mm. The most popular quena are tuned in major *G* related to the minor *E*, it have a length between 370 mm to 400 mm. Seven tonal holes are distributed on the tube; the first six are aligned with the blow end and the seventh one is located in the opposite part to the first six holes (see figure 1). The principal materials for the manufacture are the wood and the cane (zana, mamaq and bamboo).

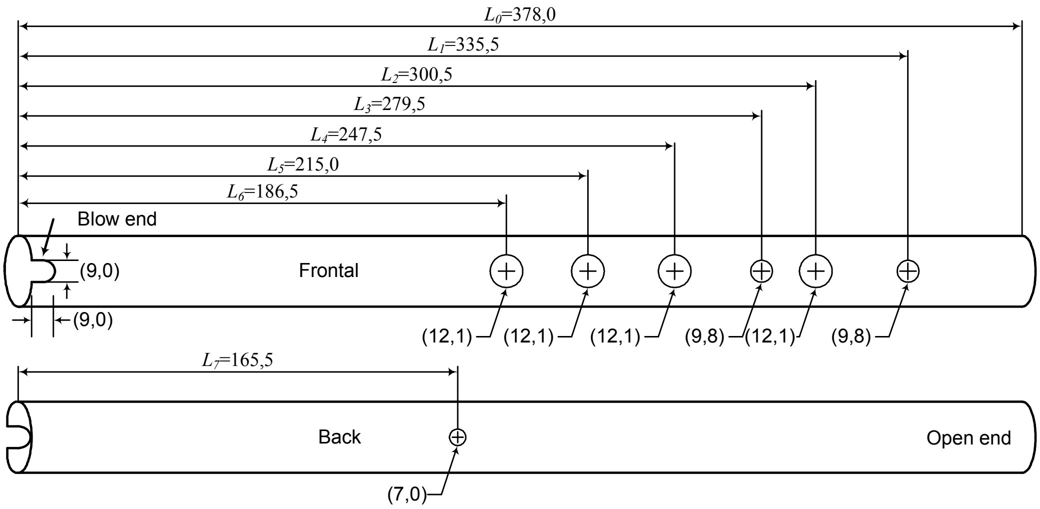

Figure 1. Scheme of the dimensional characterization of the quena.

It has been found quenas inside pre Incas tombs dispersed on the coast and the Andean of Peru, which were make by materials as cane, bone, metal and clay. The ancient quenas had around 3 to 6 tonal holes distributed in equidistant form along the tube; also, there were those of 7 tonal holes considered for the thumb finger (see figure 1). There are quenas with different blow end, the most popular blow ends have the nail shape or U shape (sillu), quadrangular shape and V shape.

END CORRECTIONS FOR THE QUENA

The quena is a Peruvian vernacular wind instrument that possesses an especial blow end and tonal holes. In this paper are studied the end corrections due to the blow end, the end corrections due to the small cavities that are formed when the tonal holes are covered, the end corrections due to the tonal holes are open and the end corrections in the open end of the duct [1].

End correction for the blow end

We must consider the existing relation between the radius of the duct and the blow end of the quena. The depth and the wide of the blow end, have the same length and this is equal to the radius of duct (see figure 2a).

A very particular characteristic of the quena is the blow end (see figure 2b), which has equivalent radius d (for this practical case). To find the end correction we consider the impedances in the duct of the blow end whose effective length is t_d and in the tube of the quena whose effective length is l_0 (both ducts are opened in both ends) [1].

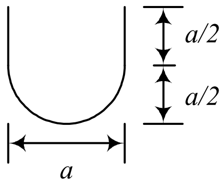

Figure 2a. Basic geometry of the quena blow end.

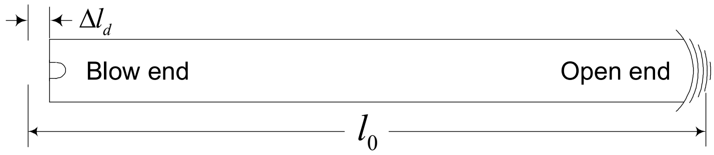

Figure 2b. Scheme of a quena that shows the blow end.

The end correction is [2]

$$\Delta l_d = (v_c/\omega)\tan^{-1}\left[\frac{\tan(\omega t_d/c)\tan(\omega l_0/c)}{(d/a)^2\tan(\omega l_0/c)+\tan(\omega t_d/c)}\right] \quad (3)$$

In the practice, this hole remains opened; in the limit of the low frequencies, it becomes independent from the frequency, also $t_e \ll l_0$. Then the Eq. (3) reduce to Eq. (4) and Eq. (5)

$$\Delta l_d = t_d\left[(d/a)^2+(t_d/l_0)\right]^{-1} \quad (4) \qquad \Delta l_d = t_d(a/d)^2 \quad (5)$$

To calculate the end correction due to the blow end we must considered the fraction β covered by the player's lips, in this case it is 11,20 % to 22,40 %. Then, the end correction due to the blow end is expressed by the Eq. (6) and the effective length due to the blow end is expressed by the Eq. (7).

$$\Delta l_d = t_d(a/d)^2(1-\beta) \quad (6) \qquad t_d = t + 1{,}45d \quad (7)$$

End correction for the tube with single tonal hole

The case of a tube with single tonal hole drilled at a distance M (see figure 3) from the open end was solved by Richardson [2,3]. The impedance for the hole has effective length t_e, and the impedance for the tube of effective length M_E are expressed by the Eqs. (8) and (9) respectively.

$$\mathbf{Z}_k = j(\rho c/\pi b^2)\tan(\omega t_e/c) \quad (8) \qquad \mathbf{Z}_0 = j(\rho c/\pi a^2)\tan(\omega M_E/c) \quad (9)$$

The characteristic equivalent impedance is given by [2]

$$\Delta l_1 = (v_c/\omega)\tan^{-1}\left[\frac{\tan(\omega t_e/c)\tan(\omega M_E/c)}{(b/a)^2\tan(\omega M_E/c)+\tan(\omega t_{E1}/c)}\right] \quad (10)$$

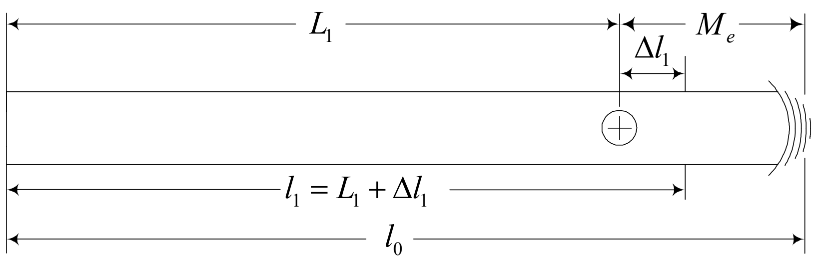

Figure 3. Scheme that shows the location of the first hole in the low end.

In the practical case M is shorter than the wavelength of the corresponding tone, when this hole remains opened it is possible to use the limit to low frequencies, then the previous Eq. becomes independent from the frequency. The end correction due to the tonal hole is expressed as

$$\Delta l_1 = t_e\left[(b/a)^2 + (t_e/M_E)\right]^{-1} \qquad (11)$$

This result is valid for any length of b and t_e.

Lateral end correction due to the formed cavity when a tonal hole is closed
The cavity that is formed when one covers a tonal hole is a small volume, whose dimensions are much smaller than the wavelength of any sound of interest [4,5]. This assumption allows that the cavity could be treated as a small mass of constant volume V that has an equivalent impedance $\mathbf{Z}_k$ (acoustics) in the open end described by Eq. (12). This cavity is attached to a duct of a tube of length l opened in both ends that have an equivalent impedance $\mathbf{Z}_e$ (specific acoustic) that is described by the Eq. (13).

$$\mathbf{Z}_k = (\pi a^2)\mathbf{Z}_t = j(\rho c^2/\omega V)(\pi a^2) \qquad (12) \qquad\qquad \mathbf{Z}_e = j\rho c \tan(\omega l/c) \qquad (13)$$

It is suitable for practical reasons, to represent both expressions in terms of the mobility

$$\mathbf{z}_k = -j\frac{\omega V}{\rho c^2(\pi a^2)} \qquad (14) \qquad\qquad \mathbf{z}_e = -j\frac{\cot(\omega l/c)}{\rho c} \qquad (15)$$

The resonance frequency will take place when the inductive reactance of $\mathbf{z}_e$ is equal in magnitude to the capacitive reactance $\mathbf{z}_k$ described by the Eq. (16). In the practical case, considering that $\cot(\omega l/c) \ll 1$, the natural frequencies of the system is described by the Eq. (17).

$$\cot(\omega l/c) = \left(\frac{V}{\pi a^2}\right)(\omega/c) \qquad (16) \qquad\qquad \omega_n = \frac{(2n-1)}{2\left[L + \Delta l_k\right]}\text{, whit } n=1,2,3,... \qquad (17)$$

Examining the Eq. (17), considering that the player finger cover almost (9/10) f of the small cavity (often it depends on the radius of the tonal hole and the wall thickness of the tube). The thickness that remains free divided is only (1/10) f, and then the end correction due to the cavity is

$$\Delta l_k = (b/a)^2(t/10) \qquad (18)$$

End correction for a tube with many tone holes
A tube with many lateral opened holes has input impedance that behavior as a small acoustic mass. This is described by the typical impedance of a cylindrical pipe with radius a, thickness t, drilled by lateral holes of radius b and a spacing $2s$ between tonal holes (see figure 4). Where t_e represents the effective length of lateral hole opened (see figure 4) [2].

$$\mathbf{Z}_0 = j(\rho c/\pi a^2)\left(\frac{1+(1/2)(a/b)^2\cot(\omega t_e/c)\tan(\omega s/c)}{1-(1/2)(a/b)^2\cot(\omega t_e/c)\cot(\omega s/c)}\right)^{1/2} \qquad (19)$$

In the practic this Eq. must be modified, because we need to know the point where will be drilled the next tonal hole, so that we must subtract the distance s between in order to obtain the desired correction [2, 3]. The Eq. (19) is altered as

$$\mathbf{Z}_0 = j(\rho c/\pi a^2)\left(\frac{1+(1/2)(a/b)^2\cot(\omega t_e/c)\tan(\omega s/c)}{1-(1/2)(a/b)^2\cot(\omega t_e/c)\cot(\omega s/c)}\right)^{1/2} - s \qquad (20)$$

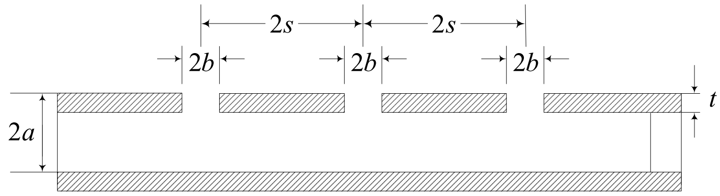

Figure 4. Scheme of a tube with many lateral opened holes [2]

We obtain the end correction as

$$\Delta l_s = (v_c/\omega)\tan^{-1}\left(\frac{(1/2)(a/b)^2\cot(\omega t_e/c)\tan(\omega s/c)+1}{(1/2)(a/b)^2\cot(\omega t_e/c)\cot(\omega s/c)-1}\right)^{1/2} - s \qquad (21)$$

The spacing between holes ($2s$) and the effective thickness t_e are much smaller than the wavelength of the musical note played in the musical instrument, for what it is possible to realize approximations as $(\omega t_e/c)\rightarrow 0$ y $(\omega t_e s/c)\rightarrow 0$, In addition $v_e = c$. Then the Eq. (21) become as

$$\Delta l_s = (s/2)\left\{\left(\left[1+4(a/b)^2(t_e/s)\right]\right)^{1/2}-1\right\} \qquad (22)$$

This end correction will serve to calculate the location of other lateral holes of the quena (after the first tonal hole), the tonal spacing (s), is not constant, then define us the difference between two physical lengths, expressed as

$$s = (1/2)(L_{(i-1)} - L_i) \qquad (23)$$

COMPUTING THE LOCATION OF THE FIRST TONAL HOLE OF THE QUENA

A musical requirement in the quena that is easily of determining is that $l_0 = 1{,}11\,l_1$. Others geometric requirement obtains of the figure 3 expressed ones in the following Eqs. [2,3].

$$l_0 = L_1 + M_E \qquad (24) \qquad\qquad l_1 = L_1 + \Delta l_1 \qquad (25)$$

Combining these equations, the result is

$$M_E = (1/2)(0{,}10 l_0)\times\left\{1+\left[1+(4/0.10)(a/b)^2(t_e/l_0)\right]^{1/2}\right\} \qquad (26)$$

If a hole of b is drilled to a distance M of the open end of the quena, the effective length is $M_E = M +$ end correction in the open end of the duct. Then

$$M = M_E - 0{,}6a \qquad (27) \qquad\qquad M = L_0 - L_1 \qquad (28)$$

Where the physical length corrected L_0 of the quena is

$$L_0 = l_0 - (\Delta l_d + 0{,}6a) \qquad (29)$$

This Eq., allows the designer to select the diameter of the tonal hole and the thickness of the tube. Then it is possible to drill the first tonal hole to a distance L_1 of the top end or to a distance M of the open end.

COMPUTING THE LOCATION OF THE TONAL REMAINING HOLES

The table 1 shows the relation that the effective lengths $l_{(i-1)}$ have with effective length l_0 for the location of every tonal hole. Those are obtained of the direct application of the Eq.: $l_i = (c/2)f$ [6].

Table 1. Effective lengths for the tonal hole locations.

Number of the length *(i=1,2,...,8)*	Musical note	Frequency f (Hz)	Effective l_i (mm)
1	*G*	396	$l_0 =$ 435,6
2	*A*	440	$l_1 =$ 392,0
3	*B*	495	$l_2 =$ 348,4
4	*C*	528	$l_3 =$ 326,7
5	*D*	594	$l_4 =$ 290,4
6	*E*	660	$l_5 =$ 261,3
7	*F#*	742,5	$l_6 =$ 232,3
8	*Gl2*	792	$l_7 =$ 217,0

When the tonal holes are opened in sequential form, the physical lengths are depended of end correction due to the blow end, end correction due the tonal hole located after the last closed hole and end correction due to the small cavity that it forms when the lateral holes are closed. A more precise calculation for the tonal hole positions from L_2 to L_7, (L_0 and L_1 already were found) can be expressed by the Eq. (30).

$$L_i = l_i - \left[\Delta l_d + \Delta l_s + \Delta l_k\right], \text{ Con } j = 2,...7 \qquad (30)$$

This Eq. will serve to calculate the location of the points where the tonal holes are going to be drilled for the follow musical notes: *B, C, Re, E, F#, G2*.

EXPERIMENTAL MODEL VALIDATION

For intentions to validate the end corrections, three quenas were constructed: one of aluminum ($a = 22$ mm, $t = 2$ mm) and two of PVC ($a = 15$ mm, $t = 3$ mm), codified like: AL1, PV1 and PV2 (see figure 5). The quenas were constructed with different diameter (b) for the tonal holes, the tune-up of each one of them has been verified. Table 2, shows the physical and tonal characteristics of the quenas (with $i = 1,2,\ldots 7$).

Figure 5. Quenas constructed based in the theoretical model, codified like: AL1, PV1 and PV2.

Table 2. Physical and tonal characteristics of the constructed quenas (AL1, PV1 and PV2).

Freq, Model (Hz)	AL1				PV1				PV2			
	b_i (mm)	L_i (mm)	Freq, (Hz)	Error (%)	b_i (mm)	L_i (mm)	Freq, (Hz)	Error (%)	b_i (mm)	L_i (mm)	Freq, (Hz)	Error (%)
396,0	closed	395,70	396,0	0,00	closed	402,7	396,3	0,08	closed	402,7	396,0	0,00
440,0	8,0	330,00	440,0	0,00	8,0	345,0	440,0	0,00	6,0	339,6	440,0	0,00
495,0	11,0	292,80	492,2	0,57	10,0	304,8	495,2	0,04	10,0	304,8	495,1	0,02
528,0	8,0	271,40	528,1	0,02	8,0	283,6	528,0	0,00	8,0	283,6	528,0	0,00
594,0	11,0	236,10	596,0	0,34	10,0	247,6	594,1	0,02	8,0	244,4	594,0	0,00
660,0	11,0	208,70	661,0	0,15	10,0	219,5	663,0	0,45	10,0	219,5	660,0	0,00
742,5	11,0	179,60	742,3	0,03	10,0	190,4	742,5	0,00	10,0	190,5	742,6	0,01
792,0	7,0	163,90	792,2	0,03	6,0	173,5	792,0	0,00	5,0	170,9	792,2	0,03

CONCLUTIONS

The quena is a very special instrument due to its origin and is unique due to the form of its blow end, still is an instrument little known by others cultures.

It has to demonstrate that it is possible to study the quena starting of simple mathematical Eqs. very known in the acoustics of musical wind instruments. An Eq. simple has settled down to calculate the equivalent diameter of blow end, as well as the determination of the edge effects

The tune-up of the quenas are very precise, which demonstrates that it is possible to construct professionals quenas of high quality with different dimensions for the tonal holes with its respective calculated locations.

The quena codified like PV2, is the one that presents better conditions of tune-up, so that we have more careful in its construction.

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